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Stochastic multi-modal transport network under demand uncertainties and adverse weather condition

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Abstract

This paper proposes a novel multi-modal transport network assignment model considering uncertainties in both demand and supply sides of the network. These uncertainties are mainly due to adverse weather conditions with different degrees of impacts on different modes. The paper provides derivations of mean and var-cov of passenger flows under the common-line framework and different dis-utility terms involved in the route/mode choice model. The model allows the risk-averse travelers to consider both an average and uncertainty of the random perceived travel time on each multi-modal path in their path choice decisions, together with the impacts of weather forecasts. The model also considers travelers' perception errors using a Probit stochastic user equilibrium framework formulated as fixed point problem. A heuristic solution algorithm is proposed for solving the fixed point problem. Numerical examples are presented to illustrate the applications of the proposed model.

Keywords
probit stochastic user equilibrium, multi-modal transport network, demand uncertainties, adverse weather condition

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Chapter

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1. Introduction

Transportation network is exposed to both demand and supply uncertainties. The causes of the supply variability can be further categorized as predictable (e.g. weather condition or scheduled road work) and less-predictable one (e.g. accident or vehicle breakdown). On the demand side, a significant level of the day-to-day demand variation can be observed in most cities. The coupling of the demand and supply uncertainties results in the recurrent variability and unreliability of the travel time and traffic condition. This issue has been gradually becoming a major concern for many countries (SACTRA, 1999).

Several stochastic network models have been proposed to incorporate demand and/or supply uncertainties into the analysis (see e.g. Clark & Watling 2005; Lo &

Tung, 2003). The main focus on modeling the capacity variability in the literature has been on the car only network with the less-predictable causes of uncertainty. With an exception, Lam et al (2008) proposes a stochastic network model which considers the effect of weather condition on the road performance. Their model also incorporates weather forecast information into the driver's route choice model. Nevertheless, their model is restricted to the auto-only network.

This paper proposes a model for representing multi-modal transport system under uncertain demand and supply. The proposed model particularly captures the effect of weather condition (predictable causes) on road performances and traveler's route/mode choice behaviors. The adverse weather deteriorates supply of different modes with various scales. The rainfall or snowfall can significantly reduce the free-flow speed and capacity of the road which also affects journey time and waiting time of bus services. On the other hand, the weather condition may not have a significant impact on a weather-proof transit system, e.g. underground network. It is plausible to assume that travelers generally do not desire to walk under such severe weather condition.

The proposed model in this paper considers auto, bus, rail, underground, and walking as possible travel modes in which a multi-modal trip is allowed. The stochastic origin-destination (OD) travel demand is assumed to follow Poisson distribution. The common-line framework for transit network modeling (Spiess & Florian, 1989) is implemented in the proposed model to represent travelers' strategies in transit network. The common-line framework requires a very complex analysis to derive the analytical property of the model (unlike the stochastic model of the auto-only network). Sections 2 and 3 present statistical properties (mean and variance) of the flows and travel dis-utility terms of the stochastic multi-modal network model.

In Hong Kong, there is a rain storm warning system with three levels signal based on the forecasted level of hourly average rainfalls. In Japan or other cold regions, the forecast of heavy snowfall is also provided to the public. The weather forecast will inevitably influence traveler's behavior. Each traveler may also have her own prior belief regarding possible weather based on her past experiences given the weather forecast information. Section 4 explains the application of Bayes' Theorem to incorporate both weather forecast and prior belief into the travel's route choice model following Lam et al. (2008).

Uncertain of their actual generalized travel costs and the weather condition, the travelers may have to choose between paths with lower expected generalized travel costs but less reliable and other more reliable paths but with higher expected generalized travel costs. Such risk-averse behaviors in the context of path choice model have been confirmed by several empirical studies (see e.g. de Palma & Picard, 2005). Section 4 defines the effective generalized travel cost as a sum between the mean and standard deviation (SD) of the generalized dis-utilities on each multi-modal route. The SD term is a proxy of the variability of the travel condition on each route. Section 5 then presents the Probit Stochastic User Equilibrium (SUE) route/mode choice model (Uchida et al, 2007) and a path-based solution algorithm for solving the assignment problem. Section 6 then presents numerical tests using a hypothetical multi-modal network. The final section then concludes the paper and recommends future research issues.

2. Preliminaries and stochastic flow model

2.1 Notations and assumptions

The notations used throughout the paper are listed as follows unless specified otherwise. For notational consistency, the italic capital letters will be used to denote random variables. The italic lower-case letters will be used to denote the means of the random variables represented by the corresponding upper-case letters.

Π	Set of O-D pair
o	Origin node
d	Destination node
K_{od}	Non-empty path set between O-D pair $o-d \in \Pi$.
Q_{od}	Stochastic travel demand between O-D pair o-d with mean q_{od}
I	Stochastic weather category.
i	Realized weather category.
$\breve{p}_i$	Prior probability of weather category i provided by weather forecast.
p_i'	Posterior probability of weather category i
$\mathrm{S_a}$	Set of physical auto links
$\mathrm{S_u}$	Set of physical underground links
$\mathrm{S_w}$	Set of physical walking links
$G(\tilde{\mathrm{S}},\mathrm{N})$	A physical multi-modal transport network with N being a set of physical nodes and $\tilde{\mathrm{S}}=\mathrm{S}_a \cup \mathrm{S}_u \cup \mathrm{S}_w$ being a set of physical links
L	$\mathrm{L}=\mathrm{L}_b \cup \mathrm{L}_u \cup \mathrm{L}_{a,w}$ where L_b, L_u, and $\mathrm{L}_{a,w}$ are sets of bus, underground, and dummy lines (for auto and walking) respectively
fr_l	Nominal frequency of service of line $l \in L$ (veh/hour)
κ_l	Vehicle capacity for service of line $l \in L$

$\tilde{G} = (\mathrm{S}, \mathrm{N})$	A hyper-network with N being a set of nodes and $\mathrm{S}=\tilde{\mathrm{S}} \cup \mathrm{S}_b$ being a set of hyper-links; S_b is a set of bus hyper-links.
$\overline{\mathrm{A}}^s$	Set of attractive lines of hyper-link $s \in S$
V^s	Number of travelers on hyper link $s \in S$ with mean v^s
V_l^s	Number of travelers on hyper link $s \in S$ using line $l \in \overline{\mathrm{A}}^s$ with mean v_l^s
$\tilde{V}_l^s$	Number of travelers sharing capacity of line *l* on hyper-link *s* but not traveling on hyper-link *s*; with mean $\tilde{v}_l^s$
$\tilde{V}^s$	Number of travelers using any of line $l \in \overline{\mathrm{A}}^s$ but not on hyper-link *s*; $\tilde{V}^s = \sum_{\forall l \in \overline{\mathrm{A}}_s} \tilde{V}_l^s$; with mean $\tilde{v}_l^s$
$\hat{V}_l^s$	Number of equivalent car unit on hyper-link $s \in \mathrm{S}_a$ (inc. bus flows)
C_l^s	Number of all passengers using line $l \in \overline{\mathrm{A}}^s$ over the same route section of hyper-link *s*
F_k^{od}	Path flow on path k connecting between O-D; with mean f_k^{od}
$\overline{T}_l^s$	Travel time on auto-link $s \in \mathrm{S}_a$ with mean $\overline{t}_l^s$
$\tilde{T}_l^s$	Perceived in-vehicle travel time with crowding effect (*PIT*) for travelers on line $l \in \overline{\mathrm{A}}^s$ over hyper-links $s \in \mathrm{S}_b \cup \mathrm{S}_u$ with mean $\tilde{t}_l^s$
$\hat{T}^s$	*PIT* for travelers using hyper-links $s \in \mathrm{S}_b \cup \mathrm{S}_u$; with mean $\hat{t}^s$
D_k^{od}	*PIT* for travelers using path k between O-D; with mean $\hat{t}_k^{od}$
w_s	Mean waiting time for service on hyper-link $s \in \mathrm{S}_b \cup \mathrm{S}_u$
y^s	Fare for using link $s \in \mathrm{S}_b \cup \mathrm{S}_u$

To facilitate the presentation of the essential ideas without loss of generality, the following basic assumptions are made in this paper:

A1. The travel demands between each O-D pair are assumed to follow independent Poisson distributions similar to the assumptions made in other previous studies (Clark & Watling, 2005). Other types of probability distribution of O-D travel demand have also been adopted, e.g. Lognormal distribution and Normal distribution. These probability distributions may also be realistic to approximate the random travel demand. In reality, further calibration and validation works on the basis of observed data are required to identify the most suitable distribution of the O-D travel demand.

A2. A Probit-based path choice model is adopted to reflect travelers' perception errors in path choice decisions due to its ability to represent correlation between different multi-modal routes (IIA issue).

A3. It is assumed that severe weather conditions degrade performances of road links which in turn affect travel times of auto and bus modes. The underground mode is assumed to be weather-proof. The bus frequency and the inconvenience of walking are also assumed to change with weather condition.

A4. The multi-modal network model proposed in this paper falls within the category of static model for long term planning at the strategic level. Therefore, it is assumed that travelers can acquire the weather forecast information prior to their departures. As such, travelers will not change their paths en-route.

2.2 Network representation

We consider a multi-modal transport network comprising of auto, bus, underground (and/or rail), and walking modes which is represented by $G = (\tilde{\mathrm{S}}, \mathrm{N})$. Bus services are provided on different routes in the auto network. Each bus and underground line is characterized by its service route, vehicle capacity, and nominal service frequency. To simplify the formulation, the non-public transport mode (auto and walk) is also associated with a dummy line with indefinite vehicle capacity and service frequency.

For a pair of stop/station nodes, there may be several public transport lines serving between these two nodes. The travelers can then board any first arriving bus from these lines. These possible lines are referred to as *attractive lines*. Each service line will be considered as an attractive line, if it helps reducing expected waiting times for transit services between these two nodes (Spiess & Florian, 1989). The transit passengers consider the set of attractive lines over each route section in their route choice decision. The common-line framework has been proposed to represent this travel strategy. For each pair of nodes, if there is at least one service line between them, a hyper-link will be defined to represent a travel strategy between this pair of nodes. Each hyper-link involves a set of attractive lines.

Given $G = (\tilde{\mathrm{S}}, \mathrm{N})$ and L a hyper-network $\tilde{G} = (\tilde{\mathrm{S}} \cup \mathrm{S}_b, \mathrm{N})$ can be defined where S_b is a set of hyper-links representing bus services between different pairs of nodes. Fig. 1 shows an example of a primitive multi-modal network with one underground line (line 1) and three bus lines (lines 2-4). Bus line 3 traverses through the lower route of the road network whereas the other two are on the upper route. Based on Fig. 1 a hyper-network, shown in Fig. 2, can be constructed. In Fig. 2, each link is annotated by link number and attractive lines in the bracket. Six hyper-links for bus services are created (hyper-links 2-7). For instance, hyper-link 2 represents the direct service of bus line 2 from nodes A to X in which line 2 is included as an attractive line. On the other hand, hyper-link 6 represents the bus service between nodes B and X in which bus lines 3 and 4 are considered as attractive lines since they serve between nodes B and X.

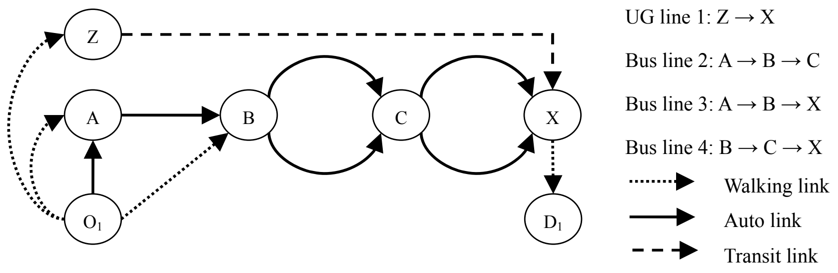

Fig. 1. Illustrative primitive physical network

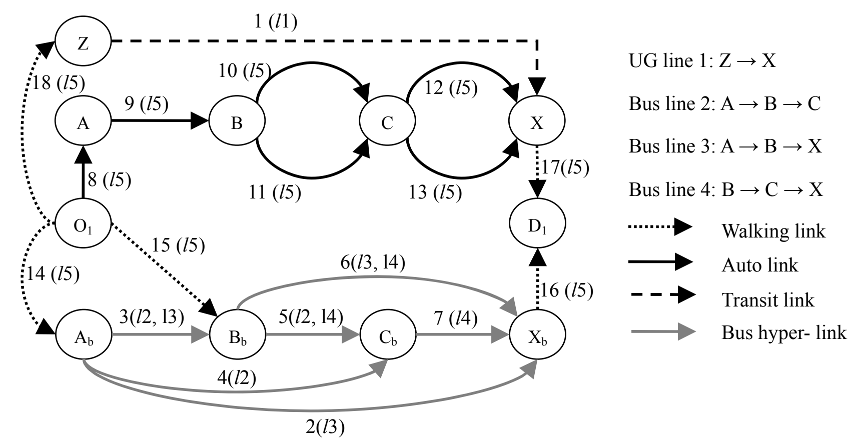

Fig. 2. Hyper-network of the illustrative example

Note that the elastic travel demand can be incorporated into the model formulation by introducing of one pseudo link for each OD pair representing the non-travel option following Connors et al (2007).

2.3 Stochastic passenger flow distribution

Following **A1**, the random O-D travel demand follows Poisson distribution, i.e. $Q_{od} \sim Poi(q_{od})$ with the mean and variance of q_{od}. Conditional on the number of traveler realized on a given day, a random traveler is assumed to choose independently between alternative multi-modal routes $k \in \mathrm{K}_{od}$ with a constant probability p_k^{od}. Following Clark & Watling (2005), the random path flow will then follow a compound distribution which is a Poisson distribution with mean (and variance) of $p_k^{od} q_{od}$, i.e. $F_k^{od} \sim Poi(p_k^{od} q_{od})$. Let $\delta_{s,k}$ denotes a dummy variable in which $\delta_{s,k} = 1$ if path k uses hyper-link s and $\delta_{s,k} = 0$ otherwise. Based on the structure of the hyper-network, the hyper-link flow can be defined as:

$$V^s = \sum_{od \in \Pi} \sum_{k \in \mathrm{K}_{od}} \delta_{s,k} \cdot F_k^{od} \,. \tag{1}$$

Using the central limit theorem, the vector of hyper-link flows follows a Multivariate Normal distribution (MVN) with the mean of:

$$E\left[V^s\right] \equiv v^s = \sum_{od \in \Pi} \sum_{k \in \mathrm{K}_{od}} \delta_{s,k} \cdot f_k^{od} = \sum_{od \in \Pi} \sum_{k \in \mathrm{K}_{od}} \delta_{s,k} \cdot p_k^{od} \cdot q_{od} \,, \tag{2}$$

and the variance-covariance (var-cov) of:

$$\mathrm{cov}\left(V^{s_i}, V^{s_j}\right) = \sum_{od \in \Pi} \sum_{k \in \mathrm{K}_{od}} \delta_{s_i,k} \cdot \delta_{s_j,k} \cdot f_k^{od} = \sum_{od \in \Pi} \sum_{k \in \mathrm{K}_{od}} \delta_{s_i,k} \cdot \delta_{s_j,k} \cdot p_k^{od} \cdot q_{od} \,. \tag{3}$$

Let $\mathbf{v}$ and Σ^V be the vector of mean and var-cov matrix of the hyper-link flows respectively. Thus, the multivariate random hyper-link flows can be defined as:

$$\mathbf{V} \sim \mathrm{MVN}\left(\mathbf{v}, \Sigma^V\right). \tag{4}$$

Based on the common-line framework, the passengers travelling on each hyper-link s will be assigned to different attractive line $l \in \bar{\mathrm{A}}^s$ as follows:

$$V_l^s = \left(\frac{fr_l}{\sum_{\tilde{l} \in \bar{\mathrm{A}}^s} fr_{\tilde{l}}} \right) \cdot V^s \,. \tag{5}$$

The term $fr_l / \sum_{\tilde{l} \in \bar{\mathrm{A}}^s} fr_{\tilde{l}}$, denoted by η_l^s, is the ratio of the frequency of line l to the total service frequency of hyper-link s. Thus, V_l^s follows a MVN with the mean:

$$E\left[V_l^s\right] = \eta_l^s \cdot E\left[V^s\right] = \eta_l^s \cdot v^s \,, \tag{6}$$

and the var-cov of:

$$\mathrm{cov}\left(V_{l_i}^{s_a}, V_{l_j}^{s_b}\right) = \min\left\{\eta_{l_i}^{s_a} \,\mathrm{cov}\left(V^{s_a}, V^{s_b}\right), \eta_{l_j}^{s_b} \,\mathrm{cov}\left(V^{s_a}, V^{s_b}\right)\right\}. \tag{7}$$

The traveler flows on different hyper-links may share the same service capacity due to possible overlapping of their sets of attractive lines and route sections. For instance, from Fig. 2 the passengers using line l_4 on hyper-link 6 share the vehicle capacity with the passengers using the same line but on hyper-link 7 over the section between nodes C and X in the physical network. This interaction influences the calculation of the in-vehicle congestion. Let $\hat{\mathrm{S}}(s)$ be a set of hyper-links (except hyper-link s) with an overlapping route section with hyper-link s in the physical network. The passengers travelling on service line l on $\tilde{s} \in \hat{S}(s)$ can then be defined as:

$$\tilde{V}_l^s = \sum_{\hat{s} \in \hat{\mathrm{S}}(s)} V_l^{\hat{s}} \ , \tag{8}$$

in which the mean of $\tilde{V}_l^s$ is:

$$E\left[\tilde{V}_l^s\right] = \sum_{\hat{s} \in \hat{\mathrm{S}}(s)} E\left[V_l^{\hat{s}}\right] = \sum_{\hat{s} \in \hat{\mathrm{S}}(s)} v_l^{\hat{s}} \ . \tag{9}$$

Thus, the total number of passenger using line $l \in \overline{\mathrm{A}}^s$ over the same route sections as hyper-link s (denoted by C_l^s) can be defined as.:

$$C_l^s = V_l^s + \tilde{V}_l^s \ . \tag{10}$$

The mean and var-cov of C_l^s can then be defined as:

$$E\left[C_l^s\right] \equiv c_l^s = E\left[V_l^s + \tilde{V}_l^s\right] = \eta_l^s \cdot v^s + \sum_{\hat{s} \in \hat{\mathrm{S}}(s)} \eta_l^{\hat{s}} \cdot v^{\hat{s}} = \sum_{\hat{s} \in \hat{\mathrm{S}}(s) \cup s} \eta_l^{\hat{s}} \cdot v^{\hat{s}} \ , \tag{11}$$

$$\mathrm{cov}\left(C_{l_i}^{s_a}, C_{l_j}^{s_b}\right) = \sum_{m \in \hat{\mathrm{S}}(s_a) \cup s_a} \ \sum_{n \in \hat{\mathrm{S}}(s_b) \cup s_b} \mathrm{cov}\left(V_{l_i}^m, V_{l_j}^n\right). \tag{12}$$

The cov between C_l^s and V_l^s , which is required for further analysis, can be expressed as:

$$\mathrm{cov}\left(V_{l_i}^{s_a}, C_{l_j}^{s_b}\right) = \sum_{n \in \hat{\mathrm{S}}(s_b) \cup s_b} \mathrm{cov}\left(V_{l_i}^{s_a}, V_{l_j}^n\right). \tag{13}$$

3. Formulation of stochastic generalized hyper- path travel cost distribution

3.1 Generalized hyper-path cost terms

The travelers will choose their multi-modal routes in the hyper-network by considering (i) perceived in-vehicle travel time (with in-vehicle congestion effect for transit modes), (ii) walking time, (iii) waiting time, and (iv) fare. Thus, the generalized travel cost for hyper-route *k* under each weather condition *i* can be defined as:

$$\begin{aligned} D_k^{od}(i) = & \sum_{\forall s \in S_a} \delta_{s,k} \cdot \hat{T}^s(i) + \sum_{\forall s \in S_b \cup S_u} \delta_{s,k} \cdot \left(\hat{T}^s(i) + \left(\frac{\theta_{wait}}{\sigma} \cdot w^s(i) \right) + \frac{y^s}{\sigma} \right) \\ & + \frac{\theta_{walk}}{\sigma} \sum_{\forall s \in S_w} \delta_{s,k} \cdot \hat{T}^s(i) \end{aligned}, \qquad (14)$$

where θ_{wait} and θ_{walk} are monetary values of waiting and walking times respectively; σ is monetary value of travel time; and w^s is waiting time. The first term in (14) is the in-vehicle travel time for the route sections of auto mode. The second term combines perceived in-vehicle travel time with in-vehicle congestion effect (referred to as *PIT*), waiting time, and fare for transit sections. The last term is the walking time.

3.2 Road travel time and effect of weather condition

Following **A3**, rainfall or snowfall may degrade the road performance due to the changes on the driving condition (e.g. reduced visibility and pavement friction). This is equivalent to a reduction in traffic parameters (e.g. lower free-flow speeds and capacities). It is possible, then, to define and calibrate the actual functional relationship between these effects and changes in different parameters of the travel time functions. To capture the weather effects and the supply uncertainty, a Generalized Bureau of Public Roads (GBPR) travel time function proposed by Lam et al (2008) is adopted:

$$\bar{T}_l^{s \in S_a}(i) = g_{t_s^0}(i) t_s^0 + \beta_s \frac{1}{g_{K_s}(i) K_s} \left(\hat{V}_l^s \right)^z \qquad \forall s \in S_a , \qquad (15)$$

$$\hat{V}_l^s = \sum_{l' \in \psi(s)} fr_{l'}(i) \cdot pce_b + \frac{V_l^s}{o_a}, \tag{16}$$

where $fr_l(i)$ is the frequency of the service line l under weather condition i; $\psi(s)$ is a set of bus lines passing $s \in \mathrm{S}_a$; pce_b is the passenger car equivalent unit for bus vehicle; o_a is the average car occupancy; K_s is the link capacity; t_s^0 is the free-flow travel time; β_s and z are parameters in the conventional BPR function; $g_{t_s^0}(i)$ and $g_{K_s}(i)$ are the scaling functions on free-flow travel time and link capacity under weather condition i respectively. Under normal weather condition ($g_{t_s^0}(0) = 1$ and $g_{K_s}(0)$ =1), the GBPR becomes the standard BPR function. This GBPR function is only applied to the auto hyper-links $s \in \mathrm{S}_a$. Since auto hyper-link is only associated with a dummy service line, the superscript l in (15) is, in fact, unnecessary and will be omitted, i.e. $\overline{T}_l^s \equiv \overline{T}^s \ \forall s \in \mathrm{S}_a$. For auto mode, there is no in-vehicle congestion effect (unlike public transport modes), in which $\hat{T}^s = \overline{T}^s \quad \forall s \in \mathrm{S}_a$.

3.3 Perceived in-vehicle travel time with crowding effect (PIT)

For bus users, the inconvenience or dis-utility from the travel time on a certain route section or hyper-link also depends on the crowding level of the vehicle. The *PIT* on hyper-link $s \in \mathrm{S}_b$ can be calculated from (i) travel times on the set of physical links related to hyper-link s and (ii) bus crowding level over that route section. The combined effect is represented by the product of the scale of in-vehicle congestion and the actual in-vehicle travel time following Fernandez et al (1994). Let $\Delta(s,l)$ be the set of the auto hyper-links used by service route of line l over the section related to hyper-link $s \in \mathrm{S}_b$. For instance, in Fig. 2 $\Delta(6,4) = \{10,12\}$ since l_4 uses auto hyper-links 10 and 12 over the route section of hyper-link 6. The *PIT* of line l over $s \in \mathrm{S}_b$ is:

$$\tilde{T}_l^s(i) = \sum_{n \in \Delta(s,l)} \left\{ \overline{T}^n(i) \cdot \left(1 + \varpi \left(\frac{C_l^s}{fr_l(i) \cdot \kappa_l} \right)^{\gamma} \right) \right\}, \tag{17}$$

where ϖ and γ are parameters of the in-vehicle congestion effect. Each hyper-link $s \in \mathrm{S}_b$ involves a number of attractive lines. Thus, the *PIT* over hyper-link s is calculated from a weighted average of all $\tilde{T}_l^s \;\; l \in \bar{\mathrm{A}}^s$:

$$\hat{T}^s(i) = \sum_{l \in \bar{\mathrm{A}}^s} \eta_l^s \cdot \tilde{T}_l^s(i). \tag{18}$$

Note that (17) and (18) are also applicable to the case of underground hyper-link in which $\bar{T}^n$ in (17) is replaced by a constant travel time of the underground hyper-link.

3.4 Waiting and walking time with the weather effect

For the transit modes, the other dis-utility term is the waiting time. For simplicity, the waiting time on a certain hyper-link $s \in \mathrm{S}_b \cup \mathrm{S}_u$ is assumed to follow:

$$w^s(i) = \frac{1}{\sum_{l \in \bar{\mathrm{A}}^s} fr_l(i)}. \tag{19}$$

The frequency of bus service may decrease under severe weather condition due to longer travel time. The formulation of waiting time can be further generalized to take into account the number of passengers in determining the waiting time (Fernandez et al 1994) and also the elastic line frequency as proposed by Lam et al. (2002).

For the walking time, the nominal walking time on each $s \in S_w$ is constant which is denoted by $\bar{T}^s$. However, under severe weather (heavy rain or snow), travelers may perceive more inconveniences from walking as compared to walking under normal weather condition. Thus, the perceived walking time on hyper-link $s \in S_w$ can be defined as:

$$\hat{T}^s(i) = \varsigma(i)\bar{T}^s, \tag{20}$$

where $\varsigma(i) \geq 1$ is the scaling function for the perceived walking time under weather condition *i*.

3.5 Stochastic generalized travel cost distribution

The distribution of the generalized travel cost in (14) under each weather scenario can be characterized by its mean and variance which can be defined as:

$$
\begin{aligned}
E\left[D_k^{od}(i)\right] &= \sum_{\forall s\in \mathrm{S}_a}\delta_{s,k}\cdot\hat{t}^s(i)+\sum_{\forall s\in \mathrm{S}_b\cup \mathrm{S}_u}\delta_{s,k}\cdot\left(\hat{t}^s(i)+\left(\frac{\theta_{wait}}{\sigma}\cdot w^s(i)\right)+\frac{y^s}{\sigma}\right)\\
&\quad+\frac{\theta_{walk}}{\sigma}\sum_{\forall s\in \mathrm{S}_w}\delta_{s,k}\cdot\hat{t}^s(i)\\
Var\left[D_k^{od}(i)\right] &= \sum_{\forall s\in \mathrm{S}}\delta_{s,k}\cdot Var\left[\hat{T}^s(i)\right]\\
&\quad+\sum_{\forall s_i\in \mathrm{S}}\sum_{\forall s_i\in \mathrm{S}}\left(\delta_{s_i,k}\cdot\delta_{s_j,k}\cdot\mathrm{cov}\left(\hat{T}^{s_i}(i),\hat{T}^{s_j}(i)\right)\right)
\end{aligned}
\quad ,(21)
$$

In this paper, it is assumed $g_{t_s^0}(i)$ and $g'_{K_s}(i)=1/\left(g_{K_s}(i)K_s\right)$ follows Normal distribution due to supply uncertainty, i.e.:

$$g'_{K_s}(i)\sim N\left(\mu_s^K,\left(\sigma_s^K\right)^2\right), \tag{22}$$

$$g_{t_s^0}(i)\sim N\left(\mu_s^t,\left(\sigma_s^t\right)^2\right), \tag{23}$$

For brevity we will omit the indicator to the weather condition (i) in the derivation in this section.

The mean perceived travel time for hyper-link $s\in \mathrm{S}_a\cup \mathrm{S}_u\cup \mathrm{S}_b$ can be defined as:

$$E\left[\hat{T}^s\right]=\sum_{l\in \bar{\mathrm{A}}^s}\eta_l^s\cdot\sum_{a\in\Delta(s,l)}\left\{E\left[\overline{T}^a\right]+\chi\cdot E\left[\overline{T}^a\left(C_l^s\right)^\gamma\right]\right\}, \tag{24}$$

where $\chi=\varpi/\left(fr_l\cdot\kappa_l\right)^\gamma$. The term $E\left[\overline{T}^a\right]=\overline{t}^a$ can then be expressed as:

$$\overline{t}^a=E\left[g_{t_a^0}t_a^0+\beta_a g'_{K_s}\left(\hat{V}_l^a\right)^z\right]=\mu_a^t t_a^0+\beta_a E\left[g'_{K_s}\left(\hat{V}_l^a\right)^z\right] \quad .(25)$$

By performing the Taylor-series expansion with the term $\left(\hat{V}_l^s\right)^n$ we can obtain:

$$\left(\hat{V}_l^a\right)^z=\left(\hat{V}_l^a-\hat{v}_l^a+\hat{v}_l^a\right)^z=\sum_{r=0}^{z}\frac{z!}{r!(z-r)!}\left(\hat{v}_l^a\right)^{n-r}\left(\hat{V}_l^a-\hat{v}_l^a\right)^r, \tag{26}$$

where $\hat{v}_l^a=E\left[\hat{V}_l^a\right]$. From (16), $\hat{V}_l^a=\sum_{l'\in\psi(a)}fr_{l'}\cdot pce_b+V_l^a/o_a$ which implies:

$$\hat{v}_l^a=\sum_{l'\in\psi(a)}fr_{l'}\cdot pce_b+v_l^a/o_a\ , \tag{27}$$

$$\left(\hat{V}_l^a-\hat{v}_l^a\right)=\left(\left(\sum_{l'\in\psi(a)}fr_{l'}\cdot pce_b+\frac{V_l^a}{o_a}\right)-\left(\sum_{l'\in\psi(a)}fr_{l'}\cdot pce_b+\frac{v_l^a}{o_a}\right)\right)=\frac{\left(V_l^a-v_l^a\right)}{o_a}. \tag{28}$$

By substituting (26)-(28) into (25), and replacing g'_{K_s} by $\mu_a^K+\left(g'_{K_a}-\mu_a^K\right)$ we obtain:

$$\overline{t}^{a}=\mu_{a}^{t}\cdot t_{a}^{0}+\beta_{a}\cdot\sum_{r=0}^{z}\left\{\Lambda_{r}\cdot\left(\mu_{a}^{K}E\left[\left(V_{l}^{a}-v_{l}^{a}\right)^{r}\right]+E\left[\left(g'_{K_{a}}-\mu_{a}^{K}\right)\left(V_{l}^{a}-v_{l}^{a}\right)^{r}\right]\right)\right\},\quad(29)$$

where $\Lambda_{r}=z!\left(\hat{v}_{l}^{a}\right)^{z-r}/\left(r!\left(z-r\right)!\left(o_{a}\right)^{r}\right)$. The terms $E\left[\left(V_{l}^{a}-v_{l}^{a}\right)^{r}\right]$ and $E\left[\left(g'_{K_{a}}-\mu_{a}^{K}\right)\left(V_{l}^{a}-v_{l}^{a}\right)^{r}\right]$ can be calculated by using the method proposed by Isserlis (1918), see e.g. Clark & Watling (2005), for estimating the moment of product of MVN which also requires the information in Section 2.3.

By substituting $\overline{T}^{a}=g_{t_{a}^{0}}t_{a}^{0}+\beta_{a}g'_{K_{a}}\left(\hat{V}^{a}\right)^{z}$ into (24) we can obtain:

$$E\left[\hat{T}^{s}\right]=\sum_{l\in\overline{\mathrm{A}}^{s}}\eta_{l}^{s}\sum_{a\in\Delta(s,l)}\left\{\overline{t}^{a}+\chi\left(t_{a}^{0}E\left[g_{t_{a}^{0}}\left(C_{l}^{s}\right)^{\gamma}\right]+\beta_{a}E\left[g'_{K_{a}}\left(\hat{V}^{a}\right)^{z}\left(C_{l}^{s}\right)^{\gamma}\right]\right)\right\},(30)$$

and by applying the Taylor-series expansion to $E\left[g_{t_{a}^{0}}\left(C_{l}^{s}\right)^{\gamma}\right]$ and $E\left[g'_{K_{a}}\left(\hat{V}^{a}\right)^{n}\left(C_{l}^{s}\right)^{\gamma}\right]$ we can obtain:

$$\begin{aligned}
E\left[\hat{T}^{s}\right] &= \sum_{l\in\overline{\mathrm{A}}^{s}}\eta_{l}^{s}\sum_{a\in\Delta(s,l)}\left\{\overline{t}^{a}+\chi\cdot\left(\mathrm{H}+\mathrm{M}\right)\right\}\\
\mathrm{H} &= \sum_{r_{1}=0}^{\gamma}\Lambda_{r_{1}}t_{a}^{0}\left(\mu_{a}^{t}E\left[\left(C_{l}^{s}-c_{l}^{s}\right)^{r_{1}}\right]+E\left[\left(g_{t_{a}^{0}}-\mu_{a}^{t}\right)\left(C_{l}^{s}-c_{l}^{s}\right)^{r_{1}}\right]\right)\qquad,(31)\\
\mathrm{M} &= \beta_{a}\left\{\sum_{r_{2}=0}^{z}\sum_{r_{3}=0}^{\gamma}\Lambda_{r_{2}}\Lambda_{r_{3}}\left(\begin{array}{l}\mu_{a}^{K}E\left[\left(\hat{V}^{a}-\hat{v}^{a}\right)^{r_{2}}\left(C_{l}^{s}-c_{l}^{s}\right)\right]\\ +E\left[\left(g'_{K_{a}}-\mu_{a}^{K}\right)\left(\hat{V}^{a}-\hat{v}^{a}\right)^{r_{2}}\left(C_{l}^{s}-c_{l}^{s}\right)^{r_{3}}\right]\end{array}\right)\right\}
\end{aligned}$$

where $\Lambda_{r_{1}}=\left(c_{l}^{s}\right)^{\gamma-r_{1}}\gamma!/\left(r_{1}!\left(\gamma-r_{1}\right)!\right)$, $\Lambda_{r_{2}}=\left(\hat{v}^{s}\right)^{z-r_{2}}z!/\left(r_{2}!\left(z-r_{2}\right)!\right)$, and $\Lambda_{r_{3}}=\left(c_{l}^{s}\right)^{\gamma-r_{3}}\gamma!/\left(r_{3}!\left(\gamma-r_{3}\right)!\right)$. The statistical properties of C_{l}^{s}, V^{s}, and V_{l}^{s} as derived in Section 2.3 can be used with the Isserlis' method to calculate $E\left[\left(g'_{K_{a}}-\mu_{a}^{K}\right)\left(\hat{V}^{a}-\hat{v}^{a}\right)^{r_{2}}\left(C_{l}^{s}-c_{l}^{s}\right)^{r_{3}}\right]$, $E\left[\left(C_{l}^{s}-c_{l}^{s}\right)^{r_{1}}\right]$, $E\left[\left(g_{t_{a}^{0}}-\mu_{a}^{t}\right)\left(C_{l}^{s}-c_{l}^{s}\right)^{r_{1}}\right]$, and $E\left[\left(\hat{V}^{a}-\hat{v}^{a}\right)^{r_{2}}\left(C_{l}^{s}-c_{l}^{s}\right)\right]$ in (31).

The formulation in (31) is applicable to the auto hyper-link ($s\in\mathrm{S}_{a}$) by setting the frequency of the dummy line (for auto and waling modes) to be infinity implying

$\chi = 0$ in which (31) reduces to $E\left[\hat{T}^s\right] = \sum_{l \in \bar{\mathrm{A}}^s} \eta_l^s \sum_{a \in \Delta(s,l)} \left\{\bar{t}^a\right\} = \bar{t}^s$. For the underground mode, (31) is applicable by setting $g'_{K_a} = \mu_a^K = 0$ and $g_{t_a^0} = \mu_a^t = 1$ (implying constant deterministic travel time and not affected by the weather condition) in which M = 0 and H = $\sum_{r_1=0}^{\gamma} \Lambda_{r_1} t_a^0 \left(\mu_a^t E\left[\left(C_l^s - c_l^s\right)^{r_1}\right]\right)$, and (31) reduces to:

$$E\left[\hat{T}^s\right] = \sum_{l \in \bar{\mathrm{A}}^s} \eta_l^s \sum_{a \in \Delta(s,l)} \left\{t_a^0 + \chi t_a^0 \left(\sum_{r_1=0}^{\gamma} \Lambda_{r_1} \left(E\left[\left(C_l^s - c_l^s\right)^{r_1}\right]\right)\right)\right\} \quad s \in \mathrm{S}_u \text{ .(32)}$$

To calculate (21), the derivation of the var-cov of $\hat{T}^s$ is also required. The var-cov can be defined as:

$$\operatorname{cov}\left(\hat{T}^a, \hat{T}^b\right) = E\left[\hat{T}^a \hat{T}^b\right] - E\left[\hat{T}^a\right] E\left[\hat{T}^b\right]. \quad (33)$$

$E\left[\hat{T}^a\right]E\left[\hat{T}^b\right]$ can be calculated from (31). $E\left[\hat{T}^a \hat{T}^b\right]$ can be expressed as:

$$E\left[\hat{T}^a \hat{T}^b\right] = \sum_{l_a \in \bar{\mathrm{A}}^a} \sum_{l_b \in \bar{\mathrm{A}}^b} \eta_{l_a}^a \eta_{l_b}^b \sum_{n_a \in \Delta(a,l_a)} \sum_{n_b \in \Delta(b,l_b)} \left\{\begin{array}{l} E\left[\bar{T}^{n_a} \bar{T}^{n_b}\right] + \chi^b E\left[\bar{T}^{n_a} \bar{T}^{n_b} \left(C_{l_b}^b\right)^{\gamma}\right] \\ + \chi^a E\left[\bar{T}^{n_b} \bar{T}^{n_a} \left(C_{l_a}^a\right)^{\gamma}\right] \\ + \chi^a \chi^b E\left[\bar{T}^{n_a} \left(C_{l_a}^a\right)^{\gamma} \bar{T}^{n_b} \left(C_{l_b}^b\right)^{\gamma}\right] \end{array}\right\}, \text{(34)}$$

where $\chi^a = \varpi / \left(fr_{l_a} \cdot \kappa_{l_a}\right)^{\gamma}$ and $\chi^b = \varpi / \left(fr_{l_b} \cdot \kappa_{l_b}\right)^{\gamma}$. The term $E\left[\bar{T}^{n_a} \bar{T}^{n_b}\right]$ can be reformulated as:

$$\begin{aligned} E\left[\bar{T}^a \bar{T}^a\right] \; = \; & t_{n_a}^0 t_{n_b}^0 E\left[g_{t_{n_a}^0} g_{t_{n_b}^0}\right] + t_{n_a}^0 \beta_{n_b} E\left[g_{t_{n_a}^0} g'_{K_{n_b}} \left(\hat{V}_l^{n_b}\right)^z\right] \\ & + t_{n_b}^0 \beta_{n_a} E\left[g_{t_{n_b}^0} g'_{K_{n_a}} \left(\hat{V}_l^{n_a}\right)^z\right] + \beta_{n_a} \beta_{n_b} E\left[g'_{K_{n_a}} \left(\hat{V}_l^{n_a}\right)^z g'_{K_{n_b}} \left(\hat{V}_l^{n_b}\right)^z\right] \end{aligned} \text{,(35)}$$

The terms $E\left[\bar{T}^{n_b} \bar{T}^{n_a} \left(C_{l_a}^a\right)^{\gamma}\right]$, $E\left[\bar{T}^{n_a} \bar{T}^{n_b} \left(C_{l_b}^b\right)^{\gamma}\right]$, $E\left[\bar{T}^{n_a} \left(C_{l_a}^a\right)^{\gamma} \bar{T}^{n_b} \left(C_{l_b}^b\right)^{\gamma}\right]$ in (34) can also be reformulated by multiplying $\left(C_{l_a}^a\right)^{\gamma}$, $\left(C_{l_b}^b\right)^{\gamma}$, or $\left(C_{l_a}^a\right)^{\gamma} \left(C_{l_b}^b\right)^{\gamma}$ to (35) respectively. Each term of the product of normally distributed random variables in (35) (e.g. $E\left[g_{t_{n_a}^0} g_{t_{n_b}^0} \left(C_{l_b}^b\right)^{\gamma}\right]$) and in $E\left[\bar{T}^{n_b} \bar{T}^{n_a} \left(C_{l_a}^a\right)^{\gamma}\right]$, $E\left[\bar{T}^{n_a} \bar{T}^{n_b} \left(C_{l_b}^b\right)^{\gamma}\right]$ and $E\left[\bar{T}^{n_a} \left(C_{l_a}^a\right)^{\gamma} \bar{T}^{n_b} \left(C_{l_b}^b\right)^{\gamma}\right]$ can be transformed to a product of moments of normal distributions by applying the Taylor-series expansion technique, see (26). Similar to the other derivations previously done in this section, this transformation then allows the application of the Isserlis' method to calculate these terms. Due to the limited space, the full derivations of these terms will be omitted.

(33) and (34) can then be used to define the var-cov of all hyper-links in the hyper-network. With the mean and var-cova of the hyper-link *PIT*s, the mean and variance of the generalized *PIT* on each hyper-route, as defined in (21), can be evaluated. The probability density function (PDF) for the generalized *PIT* of hyper-route *k* under the weather condition *i*, denoted by $\Theta\left(D_k^{od}\left(i\right)\right)$, is then characterized by its mean and variance:

$$\Theta\left(D_k^{od}\left(i\right)\right) \sim \Theta\left(E\left[D_k^{od}\left(i\right)\right], Var\left[D_k^{od}\left(i\right)\right]\right). \tag{36}$$

4. Effective generalized travel time and effect of weather forecast information

4.1 Definition and effect of weather forecast information

In practice, the travelers can acquire the rainfall information via the weather forecast. In this paper, we assume that the weather forecast provides the forecast of the likelihood of different categories of the weather condition (see e.g. http://weather.yahoo.com). Table 1 shows an example of the weather forecast information provided to the travelers.

Table 1. Example of weather category, forecasted probability, and average hourly rainfall intensity

Weather categories (i)	Forecasted probability	Average hourly rainfall intensity (π_i)
No rain/light rain ($i=0$)	50%	5 mm/hr
Normal rain ($i=1$)	25%	20 mm/hr
Amber rainstorm ($i=2$)	15%	30 mm/hr
Red rainstorm ($i=3$)	5%	50 mm/hr
Black rainstorm ($i=4$)	5%	70 mm/hr

In Table 1, each possible weather category is forecasted with the probability of its occurrence ($\breve{p}_i$). $\breve{p}_i$ is the *prior probability* of weather category *i*. From Table 1 tomorrow could have no rain/light rain, normal rain, amber rainstorm, red rainstorm, or black rainstorm with the chances of 50%, 25%, 15%, 5% and 5%, respectively. Nevertheless, the weather forecast may not be accurate, and the travelers may perceive a *posterior probability* for occurrence of each weather category based on their past experiences. Let $\widehat{p}_{i/\breve{p}}$ be the conditional probability of $\breve{p}$ (a vector of $\breve{p}_i$) given weather condition i occurs. $\widehat{p}_{i/\breve{p}}$ represents the accuracy of the weather forecast according to the past experience. For example, $\widehat{p}_{i/\breve{p}}$= 60% means that in the past among 100 times of occurrence of the weather condition *i* there were 60 times that the weather forecast had predicted the information of $\breve{p}$. Then, according to the Bayes' Theorem, the posterior probability of occurrence of *i* given the weather forecast $\breve{p}$ can be defined as:

$$p_i' = \Pr\left[i/\breve{p}\right] = \frac{\widehat{p}_{i/\breve{p}}}{\sum_{l=1}^{R} \breve{p}_l \widehat{p}_{l/\breve{p}}} \breve{p}_i = \gamma \breve{p}_i , \tag{37}$$

where *R* is the total number of weather category and p_i' is the posterior probability of i; $\gamma = \widehat{p}_{i/\breve{p}} \Big/ \left(\sum_{l=1}^{R} \breve{p}_l \widehat{p}_{l/\breve{p}} \right)$ is the updating factor related to the accuracy of weather forecast. Obviously, the following conservation equation of posterior probability holds:

$$\sum_{l=1}^{R} p_l' = 1 . \tag{38}$$

Given the vector of the posterior probability, there may be possible multiple states of the network (i.e. link and path travel times). The PDF of the hyper-path generalized *PIT* under weather category i is $\Theta\left(D_k^{od}\left(i\right)\right)$ following (36). The resultant random hyper-path generalized *PIT* from different possible weather categories is then a mixture distribution with the PDF:

$$\tilde{\Theta}\left(D_k^{od}\left(I\right)\right) = \sum_{l=1}^{R} p_l' \cdot \Theta\left(D_k^{od}\left(i\right)\right), \tag{39}$$

where $\tilde{\Theta}\left(D_k^{od}\left(I\right)\right)$ is the mixture PDF of the hyper-path generalized *PIT* considering all weather categories; $D_k^{od}\left(I\right)$ denotes the mixture hyper-path generalized *PIT* on path k .

4.2 Effective travel time of generalized perceived in-vehicle travel time

The hyper-path generalized *PIT*, $D_k^{od}\left(I\right)$, is stochastic, and hence the travelers must choose their routes by trading off between the average dis-utility and variation of the dis-utility on each route. Several indicators have been proposed to include such variation of the route cost into the analysis such as late arrival penalty (Watling, 2006) or safety margin (Lo & Tung 2003). In this paper, the SD of the generalized *PIT* is adopted as the indicator of variation of hyper-path generalized *PIT*. The effective generalized *PIT* for each hyper-path can be defined as:

$$\Gamma_k^{od}\left(I\right) = E\left[D_k^{od}\left(I\right)\right] + \pi \cdot SD\left[D_k^{od}\left(I\right)\right], \tag{40}$$

where $E\left[D_k^{od}\left(I\right)\right]$ and $SD\left[D_k^{od}\left(I\right)\right]$ are the mean and standard deviation of $D_k^{od}\left(I\right)$ respectively; π is the risk-aversion parameters (the higher is the π, the more risk-prone is the traveler).

From (39), the mean of $D_k^{od}\left(I\right)$ can be defined as:

$$E\left[D_k^{od}\left(I\right)\right] = \sum_{l=1}^{R} p_l' \cdot E\left[D_k^{od}\left(i\right)\right]. \tag{41}$$

The unconditional variance of $D_k^{od}\left(I\right)$ can be defined as:

$$\begin{aligned} Var\left[D_k^{od}\left(I\right)\right] &= E\left[Var\left[D_k^{od}\left(i\right)\right]\right] + Var\left[E\left[D_k^{od}\left(i\right)\right]\right] \\ &= \sum_{l=1}^{R} p_l' \cdot \left\{Var\left[D_k^{od}\left(i\right)\right] + \left(E\left[D_k^{od}\left(i\right)\right] - E\left[D_k^{od}\left(I\right)\right]\right)^2\right\} \end{aligned}, \tag{42}$$

Thus, the effective hyper-path generalized *PIT* is:

$$\Gamma_k^{od}\left(I\right) = \sum_{l=1}^{R} p_l' \cdot E\left[D_k^{od}\left(i\right)\right] + \pi \cdot \sqrt{\sum_{l=1}^{R} p_l' \cdot \left\{\begin{matrix} Var\left[D_k^{od}\left(i\right)\right] \\ + \left(E\left[D_k^{od}\left(i\right)\right] - E\left[D_k^{od}\left(I\right)\right]\right)^2 \end{matrix}\right\}}. \tag{43}$$

5. Probit type SUE condition and solution algorithm

The traveler's attitude toward the risk is modeled in the form of effective generalized *PIT*. The travelers' perception errors due to the imperfect knowledge of network characteristics or taste variation are modeled separately (Bell & Cassir, 2002) as the perceived effective generalized *PIT* on each hyper-path:

$$\tilde{\Gamma}_k^{od}\left(I\right)=\Gamma_k^{od}\left(I\right)+\varepsilon_k^{od}\quad \forall k\in \mathrm{K}_{od};\forall od\in \Pi . \tag{44}$$

where ε_{rs}^k is the perception error associated with the hyper-path concerned. From **A2**, $\varepsilon \sim \mathrm{MVN}\left(E\left(\varepsilon\right),\Sigma_\varepsilon\right)$ where $E\left(\varepsilon\right)$ and Σ_ε are mean vector and var-cov matrix of ε. Each hyper-link is associated with an independent normally distributed perception error term in which $E\left(\varepsilon\right)$ and Σ_ε can be calculated from the network structure, see e.g. Connors et al (2007). The fixed-point condition for the Probit SUE can be defined:

$$p_k^{od}=\Pr\left(\tilde{\Gamma}_k^{od}\left(I\right)\le \tilde{\Gamma}_{k'}^{od}\left(I\right),\forall k'\in \mathrm{K}_{od}\right). \tag{45}$$

(45) is the fixed-point condition since $\tilde{\Gamma}_k^{od}\left(I\right)$ is also a function of the vector of p_k^{od}. $\tilde{\Gamma}_k^{od}\left(I\right)$ is a continuous function of the vector of p_k^{od} which guarantees the existence of a solution to (45) based on the Brower's fixed point theorem. The uniqueness of the solution can be guaranteed if $\tilde{\Gamma}_k^{od}\left(I\right)$ is strictly monotone which requires further analysis. In this paper the approximation method as proposed by Mendell & Elston (1974) is employed to calculate the hyper-path choice probability to reduce the computational time and sampling error from the Monte-Carlo simulation.

There exist a number of efficient solution algorithms for solving the traditional UE or SUE traffic assignment problems. However, most of these methods cannot be applied directly to solve (45) due to the non link-additive of the SD element in the path effective generalized *PIT*. Thus, the path-based algorithm with Method of Successive Average (MSA) is adopted in this paper. A hyper-path set is assumed given and fixed which can be based on the observation and/or interview surveys (Van Nes et al, 2008). The algorithm can be summarized as follows:

Step 0 *Initialization*: Set j = 1; define possible hyper-path set for each OD pair (denoted as K_{od}). Find a feasible hyper-path probability vector $\mathbf{p}^j$

Step 1 *Network evaluation*: Assign $F_k^{od}\sim Poi\left(p_k^{od,j}\cdot q_{od}\right)$ to the stochastic network (see Section 2.3) and then calculate $\Gamma_k^{od,j}\left(I\right)$ following Sections 3 and 4.

Step 2 *Direction finding*: Evaluate $g_k^{od,j}=\Pr\left(\tilde{\Gamma}_k^{od,j}\left(I\right)\le \tilde{\Gamma}_{k'}^{od,j}\left(I\right),\forall k'\in \mathrm{K}_{od}\right)$.

Step 3 *Check convergence*: If $\left\|\mathbf{p}^j-\mathbf{g}^j\right\|/\left\|\mathbf{p}^j\right\|\le \Xi$ or $j > j_{max}$ terminates the algorithm; Ξ is the convergence criteria and j_{max} is the maximum iteration number

Step 4 *Move*: $\mathbf{p}^{j+1}=\mathbf{p}^j+\alpha^j\cdot\left(\mathbf{g}^j-\mathbf{p}^j\right)$ where $\alpha^j=\dfrac{1}{j}$; set $j = j$ +1 and go to Step 1.

In Step 4 various choices of the step-size are available ranging from optimal step-size to a fixed step-size. For simplicity, we adopt the commonly used step-size $\alpha^j = \frac{1}{j}$ which ensures an asymptotic rate of convergence.

6. Numerical examples

The primitive physical network in Fig. 1 is adopted in the tests in which the hyper-network can be defined as shown in Fig. 2. This network was also adopted in Uchida et al (2007). There are two O-D pairs: O_1 to D_1 and A to X. The OD demands are assumed to be $Q_{o_1 d_1} \sim Poi(5000)$ and $Q_{AX} \sim Poi(1000)$. The travelers between O_1 -D_1 can use all travel modes whereas the travelers between A-X can only bus mode. Without loss of generality, o_a and pce_b as defined in (16) are both set as 1.0. The parameters for the hyper-link dis-utility terms follow the same setting as adopted in Uchida et al (2007) and will not be repeated here. The SD of the hyper-link perception error term is set to be 30% of the free flow generalized *PIT* (excluding the fare) of that hyper-link. Table 2 shows the characteristics of the service lines respectively.

Table 2. Service line characteristics

Line	Mode	Frequency: fr_l	Vehicle capacity: K_l	Mode specific constant: α^m	coefficients for disutility specific to link: π, ρ	Calibration parameters for discomfort function: β_l, γ_l
1	bus	5	100	10	1, 0.02	1, 2
2	bus	5	50	20	1, 0.02	1, 2
3	bus	5	50	20	1, 0.02	1, 2
4	bus	5	50	20	1, 0.02	1, 2
-	auto	-	-	50	1, 0.02	-
-	walk	-	-	5	1, 0.02	-

Five weather categories shown in Table 1 are adopted for the tests. Table 3 shows the five scenarios for the weather forecast which will be tested in turn. The bold figures in Table 3 highlight the most likely weather condition in each scenario. Unless specified otherwise, under weather category 1 the bus frequency is set to be at the nominal frequency as shown in Table 3. With other weather categories, all bus frequencies are assumed to reduce to 2 vehicles/hour. The inconvenience scaling factor for walking is defined as: $\varsigma(i) = 2 \cdot \pi_i$; π_i is the average hourly rainfall intensity for weather condition *i* as defined in Table 1.

Table 3. Scenarios for different weather forecast information

Scenario	$\breve{p}_1$	$\breve{p}_2$	$\breve{p}_3$	$\breve{p}_4$	$\breve{p}_5$	$\widehat{p}_{1/\bar{p}}$	$\widehat{p}_{2/\bar{p}}$	$\widehat{p}_{3/\bar{p}}$	$\widehat{p}_{4/\bar{p}}$	$\widehat{p}_{5/\bar{p}}$
S1	**80.0%**	8.0%	6.0%	4.0%	2.0%	**90.0%**	4.0%	3.0%	2.0%	1.0%
S2	8.8%	**75.0%**	8.8%	5.0%	2.5%	7.0%	**80.0%**	7.0%	4.0%	2.0%
S3	4.5%	10.5%	**70.0%**	10.5%	4.5%	4.5%	10.5%	**70.0%**	10.5%	4.5%
S4	3.5%	7.0%	12.3%	**65.0%**	12.3%	4.0%	8.0%	14.0%	**60.0%**	14.0%
S5	4.0%	8.0%	12.0%	16.0%	**60.0%**	5.0%	10.0%	15.0%	20.0%	**50.0%**

The parameters for the GBPR function are set as:

$$g'_s(i) \sim N\left(\frac{\beta_s}{\exp(0.05 \times \pi_i)}, \left(\frac{\beta_s}{2\exp(0.05 \times \pi_i)}\right)^2\right) \quad \forall s \in \mathrm{S}_a , \tag{46}$$

$$g_{t_0^s}(i) \sim N\left(\exp(0.05 \times \pi_i), 0.25\left(\exp(0.05 \times \pi_i)\right)^2\right) \quad \forall s \in \mathrm{S}_a , \tag{47}$$

where $\beta_s = t_s^0 / (K_s)^2$. The fixed seven possible hyper-paths are defined in Table 4. Each hyper-path is also associated with the main travel mode as shown in Table 4.

Table 4. Predefined hyper-path set for each OD pair

OD pair	Hyper-path no.	Hyper-link sequence
$\mathrm{O}_1 - \mathrm{D}_1$	1 (underground mode)	18-1-16
	2 (bus mode)	15-6-16
	3 (bus mode)	14-2-16
	4 (auto mode)	8-9-10-12-17
	5 (auto mode)	8-9-11-13-17
A-X	6 (bus mode)	2
	7 (bus mode)	4-7

Table 5 shows mean hyper-path flows of all seven hyper-paths under five scenarios. Two tests are compared: model with $\varsigma(i) = 2 \cdot \pi_i$ (vary case) and model with $\varsigma(i) = 1$ (fixed case). In Table 5, the bold numbers indicate the hyper-route between O1-D1 with the maximum mean flow under different scenario.

Table 5. Hyper-route mean flows under each scenario with $\varsigma(i) = 2\pi_i$ (vary) and $\varsigma(i) = 1$ (fixed)

Path	S1		S2		S3		S4		S5	
	Varied	fixed	varied	fixed	varied	fixed	varied	fixed	varied	Fixed
1	1,296	1,377	1,325	1,517	1,417	**1,621**	**1,679**	**1,920**	**2,058**	**2,307**
2	0	0	0	112	27	370	97	530	205	619
3	0	0	0	0	0	0	0	0	0	0
4	1,851	1,811	1,835	1,683	1,775	1,499	1,609	1,272	1,364	1,033
5	**1,853**	**1,812**	**1,841**	**1,688**	**1,782**	1,510	1,616	1,278	1,372	1,041
6	501	501	501	495	501	479	496	464	487	454
7	499	499	499	505	499	521	504	536	513	546

Under S1 and S2, hyper-paths 5 and 6 are the most used under both fixed $\varsigma(i)$ and varied $\varsigma(i)$ cases. Both hyper-paths are related to the auto mode implying the dominance of auto mode under S1 and S2 whose most likely weather conditions are no-rain/light-rain and normal-rain respectively. On the other hand, under S3-S5, which involve more severe raining condition, the most used hyper-paths are hyper-path 1 (underground mode). This result is intuitive since under the normal weather condition the road network can offer its best service which should generally be more convenient than public transport modes. On the other hand, under severe weather condition the variability and degradation of the road performance may shift the demand from car to underground which is a weather-proof system.

From Table 5, the bus mode is rarely used under all scenarios (particularly hyper-path 3 which has only line 2 as an attractive line). Further analysis of the result reveals the higher mean waiting time of hyper-path 3 as compared to hyper-path 2 (which has lines 3 and 4 as attractive lines). Hyper-path 2 has higher flows under more severe weather condition which may be due to the reduction of actual travel time caused by the demand shift from auto to underground modes. The result in Table 5 also shows that in general with the weather-proof walking facility (represented by the fixed $\varsigma(i)$ case) a higher demand shift to public transport (both to underground and bus) can be achieved especially under severe weather conditions.

Table 6 presents the mean and SD of the generalized *PIT* of each hyper-path. The bold number is the minimum mean generalized *PIT* for O_1-D_1 under each scenario which is always the auto mode (hyper-paths 4 and 5). In fact, as the weather becomes more severe (S3-S5), the gap between the means generalized *PIT* of the auto and underground modes increases. Despite this wider gap, the higher demand shifts from auto to underground are achieved under S3-S5 (see Table 5). The reason for the shift is the lower SD of the generalized *PIT* of the underground as compared to the auto mode (underlined numbers in Table 6). This illustrates the influence of the effective travel time function in the mode choice model.

Table 6. Hyper-route mean and SD of hyper-route generalized *PIT* under each scenario with $\varsigma(i) = 2\pi_i$ (vary) and $\varsigma(i) = 1$ (fixed)

Path	S1		S2		S3		S4		S5	
	Varied	fixed	varied	Fixed	varied	fixed	varied	fixed	varied	Fixed
1	259(29)	236(0)	449(34)	306(0)	613(53)	369(0)	994(85)	594(0)	1,497(155)	1,011(0)
2	425(26)	358(11)	496(40)	290(18)	590(78)	327(42)	939(142)	513(82)	1,370(284)	840(173)
3	1,282(28)	1,181(42)	1,015(48)	743(47)	970(100)	626(73)	1,347(181)	823(124)	1,856(363)	1,230(251)
4	**249**(42)	**213**(27)	**430**(56)	**273**(37)	**569**(100)	**307**(65)	**904**(178)	**477**(121)	**1,323**(332)	**787**(227)
5	**249**(42)	**213**(27)	**430**(56)	**273**(37)	**569**(100)	**307**(65)	**904**(178)	**477**(121)	**1,324**(332)	**787**(227)
6	1,219(44)	1,175(42)	774(49)	737(47)	607(68)	620(73)	754(110)	817(124)	1,079(215)	1,224(251)
7	1,215(43)	1,172(41)	772(48)	735(46)	604(67)	617(71)	752(109)	818(121)	1,078(213)	1,225(245)

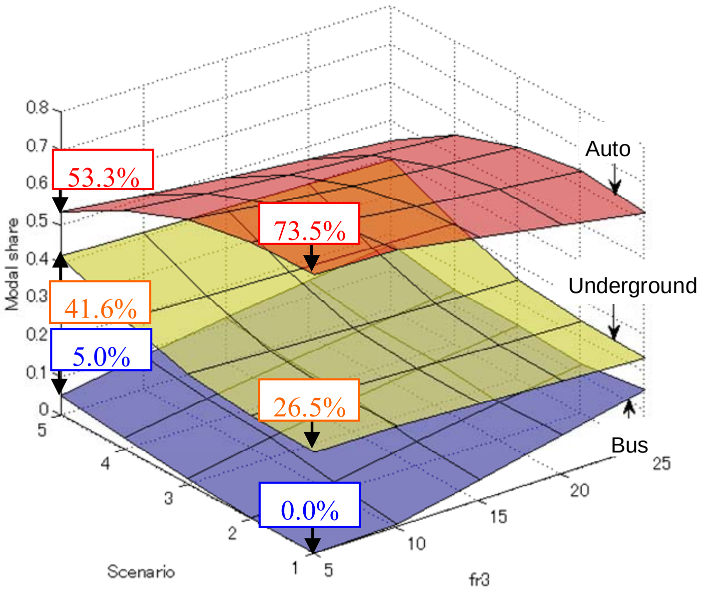

Fig. 3. Modal shares between O_1-D_1 pair

Fig. 3 shows the variations of modal shares when the frequency of bus service lines 3 increases from 5 veh/hr to 25 veh/hr under different weather scenarios. The test considers fully the scaling factor for walking and reduced bus frequency under severe weather (except the frequency of line 3). As expected, as the fr_3 increases the modal share for bus increases. Interestingly, the more severe is the weather, the higher is the modal share of bus. This is mainly due to the drastic demand shift from auto-mode. Note that the higher bus frequency also implies the higher fixed traffic volume (from bus vehicle) on the road network, i.e. $\hat{V}_l^s = \sum_{l' \in \psi(s)} fr_{l'}(i) \cdot pce_b + V_l^s / o_a$. This bus traffic becomes affects the auto travel time more under the bad weather condition closing the gap between the generalized *PIT* between bus and auto and hence attracts more demand to the bus mode (see also the result under S5 in Table 6).

7. Conclusions and discussions

This paper proposed a stochastic network model for multi-modal transport network which considers auto, bus, underground, and walking modes. The stochastic elements represent day-to-day demand variability and supply uncertainty due to adverse weather condition and random capacity degradation. The paper assumed different effects of adverse weather on the performances of different transport modes depending on their exposures to the weather condition. For the public transport network, the paper fully utilized the common-line framework to represent route choice strategy of transit passengers. The introduction of the common-line framework required a new and complex analysis (which was provided in the paper) of the statistical properties of the passenger flows and different terms involved in dis-utility function (including in-vehicle travel time, in-vehicle congestion effect, waiting time, and walking time).

The model proposed in this paper also incorporated the information from the weather forecast system into the travelers' route and mode choice models. The travelers were also assumed to have prior belief on the chances of different weather conditions given the weather forecast. To capture the trade-off between the average and variability of travel condition, the paper adopted the SD as an indicator of variability of the hyper-route travel condition in which the effective generalized travel time was defined as the sum of mean and SD of the generalized hyper-path travel time. The multi-modal hyper-path choice model was then assumed to follow the Probit SUE in which path-based solution algorithm (based on MSA) was proposed therein. The model and algorithm were tested with a hypothetical network.

The tests results were plausible and highlighted the key role of weather-proof system (e.g. underground) as the main travel mode under severe weather condition. The results also showed the need for the weather-proof walking facility and increase in bus service frequency to attract more passenger demands.

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