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Computable Urban Economic Models in Japan

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Abstract

The computable urban economic (CUE) model is a tool for analyzing real urban economies and evaluating urban polices in practice. The CUE model can output a set of variables which describe a real urban economy; a distribution of locators or activities including households and firms, a distribution of land use including residential, commercial, manufacturing, business, agricultural and other types and a distribution of land price/rent and building price/rent. The CUE model, working with transport models consistent with microeconomic theory, also output a distribution of passenger trips aggregated by OD, mode and path, a distribution of freight cargo as well. This paper first presents a general and standard form of the CUE model. The mathematical form of the CUE model and its theoretical features are described. The behavior of each economic agent including consumption, production and location choice is formalized in utility-maximizing or profit-maximizing principle. Demand and supply in land or building markets are balanced in any zone. An equilibrium state of a urban economy is defined as a solution of a system of equations and is rewritten as a solution of an equivalent mathematical programming. The paper then introduces several models in the CUE model family developed and applied in Japan. Each model is compared with other from viewpoints of experiences of application, mathematical function form and programmability of equilibrium.

Keywords

Computable Urban Economic Model, Land Use Transport Interaction Model

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ABSTRACT. The computable urban economic (CUE) model is a tool for analyzing real urban economies and evaluating urban polices in practice. The CUE model can output a set of variables which describe a real urban economy; a distribution of locators or activities including households and firms, a distribution of land use including residential, commercial, manufacturing, business, agricultural and other types and a distribution of land price/rent and building price/rent. The CUE model, working with transport models consistent with microeconomic theory, also output a distribution of passenger trips aggregated by OD, mode and path, a distribution of freight cargo as well. This paper first presents a general and standard form of the CUE model. The mathematical form of the CUE model and its theoretical features are described. The behavior of each economic agent including consumption, production and location choice is formalized in utility-maximizing or profit-maximizing principle. Demand and supply in land or building markets are balanced in any zone. An equilibrium state of a urban economy is defined as a solution of a system of equations and is rewritten as a solution of an equivalent mathematical programming. The paper then introduces several models in the CUE model family developed and applied in Japan. Each model is compared with other from viewpoints of experiences of application, mathematical function form and programmability of equilibrium.

1. INTRODUCTION

The computable urban economic (CUE) model is a tool for analyzing real urban economies and evaluating urban polices in practice. The CUE model is based on standard theories in the tradition of urban economics since Alonso *(1)* so that it can evaluate the urban polices consistently with welfare economics, in particular, with cost benefit analysis.

The CUE model is an advanced form of urban model developed in the tradition of a land use transport interaction (LUTI) model which is operational and practical. However, the LUTI model still has behavioral inconsistency and systemic inconsistency as Anas pointed out from a point of equilibrium in microeconomics *(2, 3)*. The CUE model is fully based on microeconomic foundation so as to overcome these inconsistent features in the LUTI model. The behavior of any economic agent is explicitly formalized as utility-max or profit-max and the interactions both at the inside and at the outside of markets are modeled as price-adjustment mechanism or externality.

The CUE model can output a set of variables which describe a real urban economy. The outputs in spatial dimension are categorized into two groups. The first type is a group of location specific variables; a distribution of locators or activities including households and firms, a distribution of land use including residential, commercial, manufacturing, business, agricultural and other types and a distribution of land price/rent and building price/rent. The second type is a group of flow variables; a distribution of passenger trips aggregated by origin-destination pair, by transport mode, by path or by link and node, a distribution of freight cargo as well as passenger trips. The CUE can output them by working with transport models consistent with microeconomic theory.

The CUE model consists of many equations derived from utility-max and profit-max. It uses actual economic data to analyze real urban economies and evaluate urban polices. These features and roles are the same as computable general equilibrium (CGE) model has. However, Walras' law does not hold in the CUE model while it closely does in CGE model. A variety of models with the label of the CGE model have been developed and applied to evaluation of public policies like tax reform, agreement in international trade, subsidy allocation to industrial sectors and so on. The CGE model outputs equilibrium price/quantity in all markets which are mutually consistent for cases both with and without a policy. The CGE model can estimate the net benefit of the policy in terms of equivalent variation (EV), compensating variation (CV) or consumer's surplus in Marshall-Dupuit measure (MD). Shoven and Whally contributed as a kick-off to starting the raid progress in the CGE model *(4)*. The CUE model is on a theoretical basis of the urban economic theory stylized through Alonso *(1)*, Mills *(5)*, Vickerly *(6)* and Fujita *(7)* in the same manner as the CGE is on that of the Walrasian general equilibrium theory. The CUE model can contribute to practical impact analysis for urban policies as well as the CGE can dos so in other domains of economic policy.

Urban models in the family of the CUE model have been developed and applied in Japan since the later 80's. They have been successful in analyzing impacts of urban policies in practice. In particular, the changes in land value in the impact area of a civil infrastructure project have been a great concern of a policy maker since the increment of land value should be restored to a public sector through the value capture system. The member models in the CUE model family, which explicitly describe land markets in an urban economy, have fitted to such a practical requirement in policy making.

This paper first aims at presenting a general and standard form of the CUE model. The mathematical form of the CUE model and its theoretical features are described. The behavior of each economic agent including consumption, production and location choice is formalized in utility-maximizing or profit-maximizing principle. Demand and supply in land or building markets are balanced in any zone. An equilibrium state of an urban economy is defined as a solution of a system of equations and is rewritten as a solution of an equivalent mathematical programming.

The paper then introduces several models in the CUE model family developed and applied in Japan. Each of the models is a special case of the general model with specification of indirect utility, profit, demand and supply functions, which reflect some special interests in the applications to impact analysis of urban policies. Each model is compared with others from the viewpoints of experiences of application, mathematical function form and programmability of equilibrium.

2. HISTORICAL OVERVIEW OF URBAN MODELS

Urban Economics as Theoretical Home of the CUE Model

"Modern urban economics owes its beginnings to the work of Alonso *(1)*,... Urban economics took great strides beyond Alonso's seminal contribution and became an established field through the work of Muth *(8)*, Mills *(9)*, and others." (Anas *(2)*). While those works belong to positive theory, Herbert and Stevens developed normative theory *(10)*. Fujita clarifies the theoretical relations among positive and normative theories, and extends them from static to dynamic *(7)*.

Although these studies have established the theoretical foundation of modern urban economics, they deals with continuous space and takes little account of application. Anas has applied discrete choice model to residential location and provides operational framework which is consistent with the sophisticated urban economics *(2)*.

Tradition of Land Use Transport Interaction Model

Aside from modern urban economics, developing "operational urban models" became popular, especially through the stream of quantitative geography (e.g. Foot *(11)*). Among them, Lowry had a great impact on such land use transport models *(12)*. One of the distinguished characteristics of Lowry type models are "quasi-dynamics". These operational models deal with discrete space, that is, zones since most of the available data are collected by zone. The advent of practical Geographic Information System (GIS) in '70 promoted the development of operational models. In order to compare or test a land-use/transport model with another, International Study Group on Land-Use/Transport Interaction (ISGLUTI) was set up in 1981 (Webster *(13)*). 11 organizations from 8 countries participated in the ISGLUTI study. ISGLUTI was inherited by the Special Interest Group (SIG) of the World Conference on Transport Research Society (WCTRS). Although Wegener *(14)* reviews recent developments in the field of operational LUTI models, the CUE models are not included there.

The CUE Models

In Japan, not only land-use but also land price is a matter of concern to urban and regional planners since high land price in any urban areas in Japan causes difficulties in implementation of urban policies. Urban modern economics has been attractive for urban modelers to explicitly consider land market. Consequently, after the ISGLUTI study, many land-use/transport models in Japan have been developed employing the idea by Anas *(2)*.

3. GENERAL FORM OF COMPUTABLE URBAN ECONOMIC MODEL

General Form of the CUE Model

In a variety of urban models, the CUE model is characterized by microeconomic foundation and by spatial equilibrium in tradition of urban economics. Each economic agent demands or supplies land, building, transport service and other goods at a location to choose. A footloose economic agent to be called a locator in this paper chooses the location where its utility or profit is the highest among all locations in a urban system. The land and building rent at each location (zone) which attains demand supply balancing at the location is determined simultaneously. When a urban economy is in equilibrium, the attained level of utility or profit for each type of locators is equalized among locations (zones). The CUE model can simulate a real urban economy as well as other urban models. However, it differs from them in the point that its outputs are fully consistent with benefit indicators used in practical cost benefit analysis.

Major Assumptions

The CUE model has major assumption listed in what follows.

Discrete representation of space. A spatial coverage of an urban economy is divided into zones. A zone is an area which has homogeneous geographical and economic features. A label for zone is therefore indicating a location. There exist a land market and a building market in each zone.

Locators. A footloose economic agent in the model is called a locator. The locator can choose a location where she/he consumes or produces goods.

Locators are categorized into several types. The total number of locators for each type is exogenous in the model. The model therefore descries an economy of a closed city in tradition of urban economics.

Zone specific land/ building markets and suppliers. In each zone, there exist a land market and a building market in each of which a unique equilibrium price is determined. A supplier of building in each zone is a representative developer specific to the zone. A supplier in the land market in each zone is a representative absentee landowner. Each landowner provides the land space which has homogeneous geographical and economic features. A type of land owner is thus also a label for a type of land.

The suppliers behave so as to maximize their profits. When the revenue from land or building supply has randomness, the suppliers allocate land or building space by a stochastic choice.

Location choice. Any locator demands for building space so as to maximize its utility or profit at any zone. Given a distribution of the level of indirect utility or profit among all zones, the locator chooses the zone where she/he can enjoy the highest level of indirect utility or profit. Since the distribution of the level of indirect utility or profit includes randomness, the location choice behavior is stochastic. The logit model is employed to represent a discrete choice of zone for the locator to locate.

Equilibrium. An equilibrium state of an urban economy is defined with two conditions. One is that any locator has no incentive to relocate or to change its location. In other words, the locator cannot enjoy a higher level of indirect utility or profit in other zones than in the present zone. The other condition is that demand-supply balancing or clearing in land and building markets in any zone is attained simultaneously.

Formalizing the CUE Model

Although some important symbols are defined below, a notational glossary for the CUE model is presented in the Appendix for the readers' convenience.

Locator's demand for building space. A locator maximizes her/his utility by choosing consumption of building/land space and other goods with the income constraint at a chosen location. If the locator is a firm, the utility is replaced with profit. The locator's maximization of utility or profit derives individual demand for building/land space. The demand of the locator k locating in the zone i for the building space is denoted by $q_{ik} = q(R_i, e_i, E_i, \alpha_k, Y_k)$, which is a function of the income Y_k, the building rent R_i.

A particular form of individual demand function $q_{ik} = q(R_i, e_i, E_i, \alpha_k, Y_k)$ can be derived from the corresponding indirect utility or profit function $V_{ki} = V(R_i, e_i, E_i, \alpha_k, Y_k)$, as proved in the Roy's Identity or the Hotelling's in a standard textbook of microeconomics like Varian *(15).* We therefore have to specify the functions $q_{ik} = q(R_i, e_i, E_i, \alpha_k, Y_k)$ and $V_{ki} = V(R_i, e_i, E_i, \alpha_k, Y_k)$

consistently with each other in application of the model. The endogenous geographical/economic features $e_i = e_i(N)$ and other exogenous variables E_i, α_k are included in the functions.

Location Choice Behavior. The indirect utility or profit that a locator can attain at a location or in a zone by optimizing individual building space is the attractiveness of the zone for the locator to choose. The location choice among zones is formalized with the logit model. The logit model is derived from the following maximization problem as Miyagi *(16)* and Oppenheim *(17)* had shown.

$$S(V_k, \theta_k) = \max_{a_k} \sum_{i \in \mathbf{I}_k} \left\{ a_{ki} V_{ki} - \left(\frac{1}{\theta_k} \right) a_{ki} (\ln a_{ki} - 1) \right\}, \tag{1.a}$$

$$s.t. \quad \sum_{i \in \mathbf{I}_k} a_{ki} = 1 . \tag{1.b}$$

$$S(V_k, \theta_k) = \left(\frac{1}{\theta_k} \right) \ln \left\{ \sum_{i \in \mathbf{I}_k} \exp(\theta_k V_{ki}) \right\}, \tag{2.a}$$

$$\text{and} \quad a_{ki}(V_k, \theta_k) = \frac{\exp(\theta_k V_{ki})}{\sum_{i' \in \mathbf{I}_k} \exp(\theta_k V_{ki'})} . \tag{2.b}$$

The log-sum function in (2.a), which is the maximized value by the programming in (1), is the expected value of the highest attractiveness among zones. This is a welfare measure for each type of locators.

Deterministic location choice in tradition of urban economics is a special case of (1) where θ_k is positive infinite and we can ignore the so called entropy tern $a_{ki}(\ln a_{ki} - 1)$ at the right hand side.

Demand for Land and Supply of building/land space. An aggregate supply for building space in each zone is derived from the profit of a representative developer for the zone by using the Hotelling's lemma. The building supply in zone i, $Q(R_i, P_i, Z_i, \beta)$ is derived from the profit $\pi^D(R_i, P_i, Z_i, \beta)$ as,

$$\frac{\partial \pi^D (R_i, P_i, Z_i, \beta)}{\partial R_i} = Q(R_i, P_i, Z_i, \beta) . \tag{3}$$

Land space is an input for production of building in the CUE model. An aggregate demand for land space in each zone is derived from the profit of a representative developer in the zone. The land demand in zone i, $L^D(R_i, P_i, Z_i, \beta)$ is derived from the profit $\pi^D(R_i, P_i, Z_i, \beta)$ as,

$$\frac{\partial \pi^D (R_i, P_i, Z_i, \beta)}{\partial P_i} = -L_i^D (R_i, P_i, Z_i, \beta) . \tag{4}$$

In the same manner as the supply for building, an aggregate supply for land space in each zone is derived from the profit of a landowner for the zone. The land supply in zone i, $L_i^S(P_m, W_m)$ is derived from the profit $\pi^L(P_m, W_m)$ as,

$$\frac{\partial \pi^L (P_m, W_m)}{\partial P_i} = L_i^S (P_m, W_m) . \tag{5}$$

Equilibrium. An equilibrium state of an urban economy that the CUE model describes is defined with the conditions including the distribution of locators among zones, the demand supply balancing of building space in each zone and the demand supply balancing of land space in each zone. They are formalized as follows.

Distribution of locators among zones;

$$N_{ki} = N_{kT} a_{ki} \quad for \quad all \quad i \in \{1, \cdots, I\} \quad \text{and} \quad for \quad all \quad k \in \{1, \cdots, K\}, \tag{6}$$

$$a_{ki} = \frac{\exp(\theta_k V(R_i, e_i(N), E_i, \alpha_k, Y_k))}{\sum_{i' \in \mathbf{I}_k} \exp(\theta_k V(R_{i'}, e_{i'}(N), E_{i'}, \alpha_k, Y_k))} . \tag{7}$$

Demand supply balancing of building space;

$$-\sum_{k\in\mathbf{K}} N_{kT} a(V_{k1}(\cdot)\cdots,V_{kI}(\cdot),\theta_k;i) q(R_i,e_i,E_i,\alpha_k,Y_k) + Q(R_i,P_i,Z_i,\beta) = 0 \quad for \quad all \quad i\in\{1,\cdots,I\}\ . \tag{8}$$

Demand supply balancing of land space;

$$L_i^S(P_m,W_m) - L^D(R_i,P_i,Z_i,\beta) = 0 \quad for \quad all \quad i\in\{1,\cdots,I\} \quad and \quad m\in\{1,\cdots,M\}\ . \tag{9}$$

Note that P_m is a vector while P_i is scalar.

Programmability of equilibrium

Programmability is a mathematical property that a solution of an equilibrium model is obtained as the solution of a corresponding optimization problem. The necessary conditions of the optimization problem are equivalent to a system of equations/inequalities in the equilibrium problem. That has been well-known as the equivalence between an equilibrium by the Wardrop Principle and a solution of the Beckmann problem in traffic assignment. The programmability in traffic assignment has been used for development of efficient algorithms for solution searching.

The CUE model is *programmable* in the sense above mentioned in some cases but not in all cases. If the CUE model is programmable, techniques for efficient solution search which are developed in traffic assignment can be employed for the solution search in the CUE model (*e.g.* Kim *(18)*).

The programmability of the CUE model requires two conditions. One is that the term for externality can be fixed to be a constant value as $e_i(N)=\overline{e}_i$. The other is that the marginal utility of income can also be fixed to be a constant value as $MIV(\bullet)=\overline{MIV}_k$. The former condition corresponds to no asymmetric interaction between links in traffic assignment.

The mathematical program equivalent to the equilibrium problem stated in the conditions from (6) to (9) is formalized in what follows.

$$SW(E,\overline{e},Z,W,\theta,\alpha,\beta) = \min_{R,P} \sum_{k\in\mathbf{K}} \frac{N_{kT}}{\overline{MIV}_k} \cdot S(V_k,\theta_k) + \sum_{i\in\mathbf{I}} \pi^D(R_i,P_i,Z_i,\beta) + \sum_{m\in\mathbf{M}} \pi^L(P_m,W_m)\ . \tag{10}$$

From the Roy's identity, the Hotelling's lemma and the property of the log-sum function all of which are applied forms of the envelop theorem, the necessary condition for (8) are derived as the following system of equations.

Roy's identify:

$$\frac{\partial V(\bullet)}{\partial R_i} = -\overline{MIV}_k \cdot q(R_i,\overline{e}_i,E_i,\alpha_k,Y_k)\ . \tag{11}$$

Hotelling's lemma:

$$Q_i = Q(R_i,P_i,Z_i,\beta) = \frac{\partial \pi^D(R_i,P_i,Z_i,\beta)}{\partial R_i}\ , \tag{12}$$

$$L_i^D = L^D(R_i,P_i,Z_i,\beta) = -\frac{\partial \pi^D(R_i,P_i,Z_i,\beta)}{\partial P_i}\ , \tag{13}$$

and $$L_i^S = L_i^S(P_m,W_m) = \frac{\partial \pi^L(P_m,W_m)}{\partial P_i}\ . \tag{14}$$

Property of log-sum function:

$$a_{ki} = a(V_k,\theta_k) = \frac{\partial S(V_k,\theta_k)}{\partial V_{ki}}\ . \tag{15}$$

The mathematical programming in (10) has the first order conditions as,

$$-\sum_{k\in\mathbf{K}} N_{kT} a_{ki}(V_{k1}(\cdot)\cdots,V_{kI}(\cdot),\theta_k) q(R_i,\overline{e}_i,E_i,\alpha_k,Y_k) + Q(R_i,P_i,Z_i,\beta) = 0 \quad for \quad all \quad i\in\{1,\cdots,I\}\ , \tag{16}$$

and $$L_i^S(P_m,W_m) - L^D(R_i,P_i,Z_i,\beta) = 0 \quad for\ all\ i\in\{1,\cdots,I\}\ . \tag{17}$$

They are the same ones as (8) and (9).

Since externality in an urban system is so essential in urban policies, particularly in environmental policies, the term $e_i(N)$ should not be fixed in general. The programmability is however still useful in searching solutions of the equilibrium problem stated in the conditions from (6) to (9). The equilibrium solution is obtained by repeating to solve the program in (10) in iterations with changing $e_i(N)$.

The indirect utility of locators divided by the marginal utility of income $\overline{MIV}_k$ is summed up in monetary term in the objective function as stated in (10). Following Negishi *(19)*, the sum of utility weighted with $1/MIV_k(\cdot)$ represents the social welfare function, the maximization of which results in Walrasian competitive equilibrium and therefore also in the Pareto efficient allocation. However, the marginal utility of income $\overline{MIV}_k$ in (10) is fixed to solve the mathematical program. To find the equilibrium solution in more general case that the marginal utility of income $MIV_k(\cdot)$ is fully endogenous, we have to repeat solving the mathematical program in (10) in iteration with changing $MIV_k(\cdot)$. This is the same manner for searching equilibrium as that for changing $e_i(N)$, as explained in the above.

Welfare measure and consistency with Cost Benefit Analysis

The indirect utility and profit functions in the CUE model are consistent with benefit measure like equivalent variation (EV), compensating variation (CV) or consumer's surplus in Marshall-Dupuit measure (MD). See e.g. Varian *(15)* for the definitions of these measures.

A change in social surplus formalized as the objective function in the mathematical programming in (10) is the exact measure of the social benefit of an urban policy. The social surplus is recalled from (10) to be,

$$SS = \sum_{k\in\mathbf{K}} \frac{N_{kT}}{\overline{MIV}_k} \cdot S(V_k, \theta_k) + \sum_{i\in\mathbf{I}} \pi^D(R_i, P_i, Z_i, \beta) + \sum_{m\in\mathbf{M}} \pi^L(P_m, W_m, \gamma). \tag{18}$$

Since the log-sum function of the indirect utility of a locator is divided by the marginal utility of income as $S(V_k, \theta_k)/\overline{MIV}_k$, the benefit of the locator does not differ between EV, CV and MD. The social benefit measured as the change in the social surplus is indicated in the integral form as,

$$\begin{aligned} &\Delta SS_{a\to b} \\ &= \oint_{a\to b} \sum_{k\in\mathbf{K}} \frac{N_{kT}}{\overline{MIV}_k} \cdot \left\{ \sum_{i\in\mathbf{I}_k} \frac{\partial S(V_k,\theta_k)}{\partial V_{ki}} \left(\frac{\partial V_{ki}(\bullet)}{\partial R_i} dR_i + \nabla_{e_i}(V_{ki}(\bullet)) \cdot de_i + \nabla_{E_i}(V_{ki}(\bullet)) \cdot dE_i \right) \right\} \\ &\quad + \sum_{i\in\mathbf{I}} \left\{ \frac{\partial \pi^D(R_i,P_i,Z_i,\beta)}{\partial R_i} dR_i + \frac{\partial \pi^D(R_i,P_i,Z_i,\beta)}{\partial P_i} dP_i \right\} + \sum_{m\in\mathbf{M}} \left(\sum_{i\in\mathbf{I}_m} \frac{\partial \pi^L(P_m,W_m,\gamma)}{\partial P_i} dP_i \right) \end{aligned} \tag{19}$$

where

$$\nabla_{e_i}(V_{ki}(\bullet)) \cdot de_i = \sum_{g=1}^{G} \frac{\partial V_{ki}(\bullet)}{\partial e_{ig}} de_{ig}, \tag{20}$$

$$\text{and} \quad \nabla_{E_i}(V_{ki}(\bullet)) \cdot dE_i = \sum_{g=1}^{G'} \frac{\partial V_{ki}(\bullet)}{\partial E_{ig}} dE_{ig}. \tag{21}$$

$\nabla_x(z(x,y))$ denotes the gradient vector of the function $z(x,y)$ with respect to x.

In the same manner as the mathematical programming in (10) yields to (16) and (17), the social benefit in (19) is rewritten as,

$$\begin{aligned} &\Delta SS_{a\to b} \\ &= \oint_{a\to b} \left\langle \sum_{k\in\mathbf{K}} \frac{N_{kT}}{\overline{MIV}_k} \cdot \left[\sum_{i\in\mathbf{I}_k} a_{ki}(V_k,\theta_k) \left\{ -q(R_i,e_i,E_i,\alpha_k,Y_k) + \nabla_{e_i}(V_{ki}(\bullet)) \cdot de_i + \nabla_{E_i}(V_{ki}(\bullet)) \cdot dE_i \right\} \right] \right. \\ &\quad + \sum_{i\in\mathbf{I}} \left\{ Q(R_i,P_i,Z_i,\beta) dR_i - L^D(R_i,P_i,Z_i,\beta) dP_i \right\} , \\ &\quad \left. + \sum_{m\in\mathbf{M}} \sum_{i\in\mathbf{I}_m} L_i^S(P_m,W_m,\gamma) dP_i \right\rangle \end{aligned}$$

(22)

Rearranging the terms in (22), the social benefit is furthermore rewritten as,

$$\Delta SS_{a\to b} = \oint_{a\to b}\left\langle \sum_{k\in\mathbf{K}} \frac{N_{kT}}{\overline{MIV}_k}\left[\sum_{i\in\mathbf{I}_k} a_{ki}(V_k,\theta_k)\left\{\nabla_{e_i}(V_{ki}(\bullet))\cdot de_i + \nabla_{E_i}(V_{ki}(\bullet))\cdot dE_i\right\}\right]\right. \quad , (23)$$

$$\left. + \sum_{i\in\mathbf{I}}\left[\left\{-N_{kT}a_{ki}(V_k,\theta_k)q(R_i,e_i,E_i,\alpha_k,Y_k) + Q(R_i,P_i,Z_i,\beta)dR_i\right\} + \left\{L_i^S(P_m,W_m) - L^D(R_i,P_i,Z_i,\beta)\right\}dP_i\right]\right\rangle$$

where $\sum_{k\in\mathbf{K}}\sum_{i\in\mathbf{I}_k} = \sum_{i\in\mathbf{I}}$ and $\sum_{m\in\mathbf{M}}\sum_{i\in\mathbf{I}_m} = \sum_{i\in\mathbf{I}}$ are considered.

Since the demand-supply balancing for building space stated in (8) and that for land in (9) hold in an equilibrium state of an urban economy, the terms at the inside of $\sum_{i\in\mathbf{I}}$ in (23) are cancelled out with each other. Finally the social benefit is

$$\Delta SS_{a\to b} = \oint_{a\to b}\sum_{k\in\mathbf{K}}\frac{N_{kT}}{\overline{MIV}_k}\left[\sum_{i\in\mathbf{I}_k} a_{ki}(V_k,\theta_k)\left\{\nabla_{e_i}(V_{ki}(\bullet) + \nabla_{E_i}(V_{ki}(\bullet))\cdot dE_i)\cdot de_i\right\}\right] \quad (24)$$

The final form in (24) implies that the benefit terms caused by price changes both in building markets and in land markets are excluded from the social benefit. However, from the viewpoint of a distribution of benefit, the decomposed form of social benefit in (23) shows that the developers and landowners can enjoy the change in their profit. When a Value Capture Measure like Tax Increment Financing (TIF) or Special Assessment District (SAD) which aims at restoring the benefits of the developers and landowners to financing of infrastructure investment is a great concern of urban policies, the assessment of the terms excluded from (23) is an inevitable task for a policy maker.

Since E_i denotes the exogenous geographical/economic features or exogenous attributes of zone i, their changes dE_i are interpreted as direct impacts of the policy. On the other hand, e_i denotes the endogenous geographical/economic features of zone. Their changes de_i mean the indirect impacts which are caused by propagation of the direct impacts. Terms stated in (20) or (21) with coefficient $1/\overline{MIV}_k$ imply changes in consumer's surplus of transport service, which are usually accounted in cost benefit analysis in practice.

Since the CUE model has formalized $e_i = e_i(N)$ for representing externalities, if the urban economy has installed the first best pricing or tax to internalize them like congestion charge or environmental emission tax, the indirect impacts must disappear in (19).

4. CUE MODELS DEVELOPED IN JAPAN

List of Models

In Japan, there are several types of models classified into the CUE model. They have been developed by Japanese urban modelers and applied to urban areas in Japan. We here compare the models from various viewpoints. The models are listed as below.

- **D**ouble-**S**ide **D**iscrete **C**hoice Model (**DSDC** Model) *(20, 21)*
- **D**iscrete-**C**ontinuous **L**and **D**emand Model (**DCLD** Model) *(22, 23)*
- **R**andom **U**tility / **R**ent-**B**idding **An**alysis Model (**RURBAN** Model) *(24, 25, 26)*
- **B**uilding **D**emand-**S**upply **B**alancing Model (**BDSB** Model) *(27, 28)*
- **C**ontinuous-Discrete **L**and **S**upply Model (**CDLS** Model) *(29, 30)*
- **N**eo **C**omputable **U**rban **E**conomic Model Family (**NCUE** Model Family)
  - CUE for **R**iver Improvement Project (**R-CUE**) *(32)*
  - CUE for **G**ifu Urban Area (**G-CUE**) *(32, 33)*
  - CUE by **V**alue **M**anagement Institute (**VM-CUE**) *(34, 35, 36)*

Double-Side Discrete Choice Model (DSDC Model) is characterized with the feature that not only locator's choice of a location is formalized by the logit model, but allocation of land to each locator type is also by the logit model. Both locator side (demand side of building space) and landowner side (supply side of land) are thus simultaneously modeled by the discrete choice model.

Discrete-Continuous Land Demand Model (DCLD Model) has modeled that a locator first chooses a location as a discrete zone and then determines a demand for land (building space) as a continuous variable. Two-level choice in this approach is called the Discrete-Continuous Choice. Since the model has simplified the supply side of land so as to mainly focus the demand side, the model is characterized by the Discrete-Continuous Land Demand.

Random Utility / Rent-Bidding Analysis Model (RURBAN Model) was based simultaneously both on the random utility theory and the random bidding theory in its original formulation. The original formulation contained an inconsistency with price mechanism in market equilibrium. This paper therefore reformulates the RURBAN model so as to solve the inconsistency.

Building Demand-Supply Balancing Model (BDSB Model) has been motivated by formalizing building market explicitly. The modeling of building market is the heart of analysis for emergence of high-raised buildings particularly in a city center. The model has uniquely formalized the building market.

Continuous-Discrete Land Supply Model (CDLS Model) is characterized by the two-level choice structure in land supply. The model has assumed that the landowner first determines the total amount of land supply as a continuous variable and then allocates the amount to each type of locators by the discrete choice (the logit model). The model is so unique to represent the Continuous-Discrete choice behavior of a landowner.

Neo Computable Urban Economic Model Family (NCUE Model Family) consists of the CUE models that the authors of this paper have developed. Member models in the family have been applied to a variety of urban policies including transport, land use regulation, urban redevelopment, residential area development and business district reform and so on. Such a verity of applications has required each member model in the family to together work with other simulation models like flood simulation model, CO2 emission model or transport pricing model.

Applications to Impact Analysis of Urban Policies

Table 1 compares the models from the point of application to practical policy analysis. The models have been applied to impact analysis of urban policies in medium size cities except VM-CUE which has been applied to Tokyo Metropolitan Area.

The policies targeted in impact analysis are not only transport network development plans and land use regulations at a regional master plan level but also particular projects and policies like new suburban railway, new guide-way system, new commuter railway, new road investment, flood control countermeasure, ring road development, deregulation of floor-area ratio, road pricing and railways pricing. The CUE models have been applied to impact analysis of a large variety of policies.

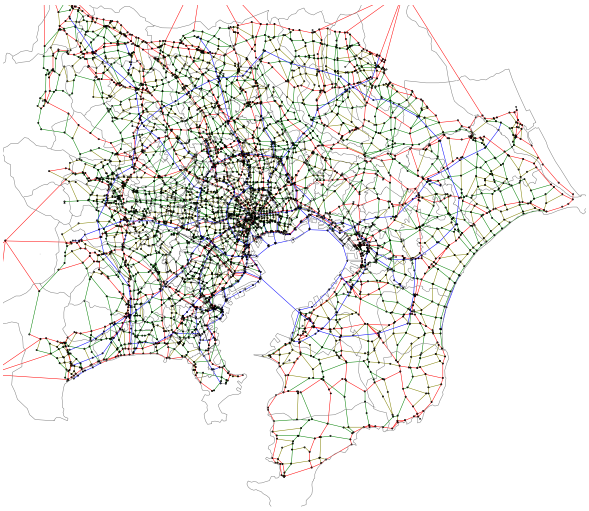
The impacts of a policy are represented as changes in distribution of locators, building rents, land rents, trips and environmental emissions. The distribution of trips can be outputted by transport network models interactively working with the CUE model. The environmental emissions are calculated by engineering models or material flow models combined with the CUE model. The impact analysis in these years inevitably has to evaluate reduction of Green House Gas emission, particularly CO2 emission. The VM-CUE in NCUE Family Model has targeted the reduction of CO2 emission from transport sectors by a variety of urban policies in Tokyo Metropolitan Area (Figures 1-3 and Table 2).

TABLE 1: Comparison of Application

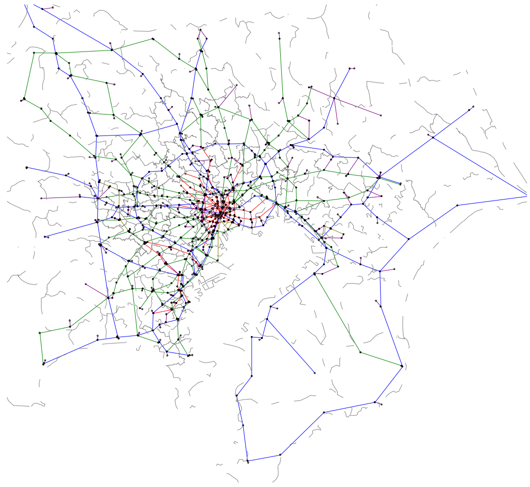
MODEL		Paper	Case Study Area(s)				Target Policy
			City/Area	Area (km^2)	Population (thousands)	Number of Zones / Mesh Size	
Double-Side Discrete Choice Model (DSDC Model)		Hayashi and Doi *(19)*	Nagoya M. A.	2,170	c.a. 5,400	12	New suburban railway
		Hayashi and Tomita *(20)*	Nagoya M. A.	2,170	c.a. 5,400	14	Transport improvement
Discrete-Continuous Land Demand Model (DCLD Model)		Morisugi, Ohno and Miyagi *(22)*	Gifu City	1,315	1,264	12	Road network expansion
Random Utility / Rent-Bidding Analysis Model (RURBAN Model)		Miyamoto and Kitazume *(23)*	Sapporo M. A.	c.a. 1,100	c.a. 1,500	1km by 1km	Transport investment
		Miyamoto, Noami, Kuwata, and Yokozawa *(24)*	Sapporo M. A.	c.a. 1,000	c.a. 1,600	1km by 1km	Transport network development, and Land use regulations
		Miyamoto, Vichiensan, Sugiki and Kitazume *(25)*	Sapporo M. A.	3,348	2,323	8,025	Subway line extension
Building Demand-Supply Balancing Model (BDSB Model)		Ueda, Hiratani and Tsutsumi *(26)*	Hiroshima City	740	1,086	8	New guide-way system
		Ueda, Tsutsumi and Nakamura *(27)*	Northeast Tokyo M. A.	c.a. 1,900	c.a. 2,300	12	New commuter railway
Continuous-Discrete Land Supply Model (CDLS Model)		Yoon, Aoyama, Nakagawa and Matsunaka *(29)*	Kyoto City and a neighboring prefecture	4,628	2,751	65	New Road Construction
Neo Computable Urban Economic Model Family (NCUE Model Family)	CUE for River Improvement Project (R-CUE)	Takagi and Ueda *(30)*	Sakai River basin (in Gifu City)	60	66	1km by 1km	Flood control countermeasure
	CUE for Gifu Urban Area (G-CUE)	Takagi, Muto and Ueda *(31)* Muto, Takagi and Ueda *(32)*	Gifu City	252	469	47	Ring road
	CUE by Value Management Institute (VM-CUE)	Muto, Ueda, Yamaguchi and Yamasaki *(33)* Yamasaki, Ueda and Muto *(34)* Yamasaki, Ueda and Muto *(35)*	Tokyo M. A.	15,000	34,860	197	3-ring roads, Relaxation of floor-area ratio, Road pricing, Reduction of railways fares, Railway Improvement

M.A.: Metropolitan Area

(a) Coverage Area (197 zones)

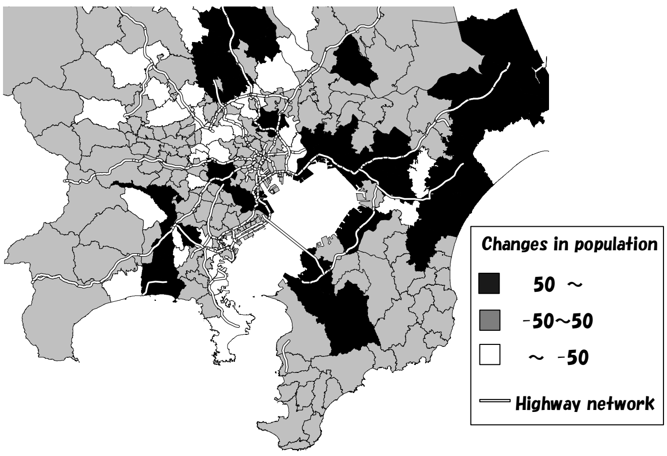


(b) Road Network

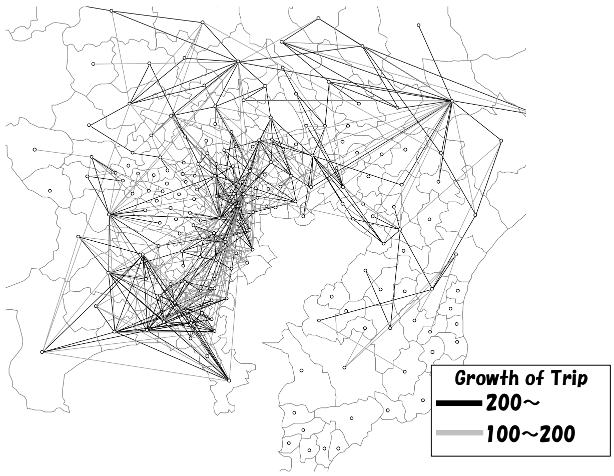


(c) Railway Network

FIGURE 1: Coverage Area and Transport Network in VMcue



(a) Population change
[Reducing the fare on TOKYO WAN AQUA-LINE]



(b) Change in trip by car
[Construction of 3-Ring Road]

FIGURE 2: An example of the Outputs by VMcue

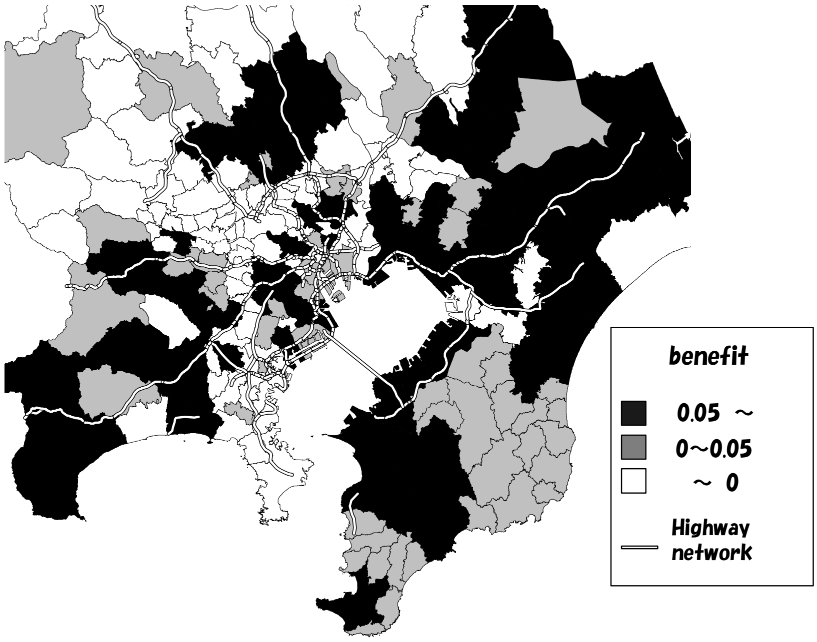


FIGURE 3: Another example of the outputs by VMcue:
Project benefit in the incidence form [Reducing the fare on Tokoyo Bay Aqua Line]

TABLE 2: Compendium of available outputs by VMcue

socioeconomic indicator	gross regional product
	land rent
	household income
	leisure time
	commuters unable to get home
	CO_2emission
	NO_xemission
Landuse indicator	population
	Employee
	Land supply by the absentee landlord
	lot area
Transport indicator	Trip generation
	traffic modal split(Road or Railway)
	Traffic Volume of Link
	generalized cost
	congestion
Evaluation indicator	Benefit（household，Firm，landlord）

Zone Setting

Table 3 compares a style of model zone setting.

There are two groups from the point of zone setting.

Zone defined for each pair of locator type and land type. A zone is defined for a pair of locator type $k \in \mathbf{K} = \{1, \cdots, K\}$ and land type $m \in \mathbf{M} = \{1, \cdots, M\}$. A label of zone $i \in \mathbf{I}$ means $i = (k, m) \in \{(k,1), \cdots, (K, M)\}$. A locator in type $k \in \mathbf{K}$ can choose only the zones belonging to the subset $\mathbf{I}_k = \{(k,1), \cdots, (k, M)\} \subseteq \mathbf{I}$. An amount of land in type $m \in \mathbf{M}$ can be supplied or allocated to the zones belonging to $\mathbf{I}_m = \{(1,m), \cdots, (K,m)\}$.

DSCE model, RURBAN model and CDLS model have employed this style of zone setting.

Zone defined for each land type. A zone is defined for each land type $m \in \mathbf{M} = \{1, \cdots, M\}$. A label of zone means $i = m \in \{1, \cdots, M\}$. Since a locator in any type $k \in \mathbf{K}$ can choose any zones in an urban economy, then the choice set of zones for a locator to locate is written as $\mathbf{I}_k = \{1, \cdots, M\} = \mathbf{I}$. A landowner in type $m \in \mathbf{M}$ can supply the land only to the zone labeled by $i = m \in \mathbf{M} = \{1, \cdots, M\}$ and the set of zones for the landowner to supply is $\mathbf{I}_m = \{m\} \subseteq \mathbf{I}$. In the zone setting, locators in different types can locate in the same zone simultaneously.

DCLD Model, BDSB Model and NCUE Model Family have employed the above style of zone setting.

TABLE 3: Comparison of Zone Setting

MODEL	Label for zone	Label of zone for locator to locate	Label of zone for landowner to supply
General Form	$i \in \mathbf{I}$	$\mathbf{I}_k$	$\mathbf{I}_m$
DSDC Model	$i = (k, m) \in \mathbf{I} = \{(1,1), \cdots, (K, M)\}$	$\mathbf{I}_k = \{(k,1), \cdots, (k, M)\}$	$\mathbf{I}_m = \{(1,m), \cdots, (K,m)\}$
DCLD Model	$i = m \in \mathbf{I} = \{1, \cdots, M\}$	$\mathbf{I}_k = \{1, \cdots, M\}$	$\mathbf{I}_m = \{m\}$
RURBAN Model	$i = (k, m) \in \mathbf{I} = \{(1,1), \cdots, (K, M)\}$	$\mathbf{I}_k = \{(k,1), \cdots, (k, M)\}$	$\mathbf{I}_m = \{(1,m), \cdots, (K,m)\}$
BDSB Model	$i = m \in \mathbf{I} = \{1, \cdots, M\}$	$\mathbf{I}_k = \{1, \cdots, M\}$	$\mathbf{I}_m = \{m\}$
CDLS Model	$i = (k, m) \in \mathbf{I} = \{(1,1), \cdots, (K, M)\}$	$\mathbf{I}_k = \{(k,1), \cdots, (k, M)\}$	$\mathbf{I}_m = \{(1,m), \cdots, (K,m)\}$
NCUE Model Family	$i = m \in \mathbf{I} = \{1, \cdots, M\}$	$\mathbf{I}_k = \{1, \cdots, M\}$	$\mathbf{I}_m = \{m\}$

Location Attractiveness Function and Individual Demand Function for Building

Location attractiveness function and individual demand function for building space are compared in the second and third columns in Table 4 respectively.

Price-elastic demand. The log-linear indirect utility function $V_{ki} = \phi(e_i, E_i, \alpha_k) - \rho_k \ln R_i + \varphi_k \ln Y_k$ must yield to the individual demand function for building space $q_{ik} = \rho_k \dfrac{Y_k}{R_i}$, as employed in DCLD Model, RURBAN Model and NCUE Model Family. In contrast, BDSB Model has transformed the linear function of individual demand for building space $q_{ik} = a - bR_i$ into the indirect utility function as an integral $\int_{R_i}^{\bar{R}} (a - bs) ds$. The integral indicates the consumer's surplus and then the indirect utility is measured in monetary term.

Price-inelastic demand. DSDC Model and CDLS Model have treated an individual demand for building space as an exogenous parameter in static equilibrium. If the demand is price-inelastic in real urban economy, this would simplify an analysis in practice.

Aggregate Supply for Building and Demand for Land

Aggregate supply for building and demand for land are compared in the fourth and fifth columns in Table 4 respectively.

Endogenous building supply. Only BDSB Model has described endogenous supply of building space by a representative developer for each zone. The land is an input factor for producing building space. The supply function for building $Q_i = \beta_0 \beta_1 R_i^{\beta_1 - 1} P_i^{\beta_1 - 1}$ and the factor demand function for land $L_i^D = \beta_0 \beta_2 R_i^{\beta_1} P_i^{-\beta_2 - 1}$ are derived from the developer's profit maximization with the Cobb-Douglas production function $Q = \beta_0 (L^D)^{\beta_1} (Cap)^{1-\beta_1}$, where *Cap* denotes a capital input for producing building space. The capital rent is normalized to be one.

Exogenous building supply. In other models in the CUE model family, it is assumed that the aggregate supply of building space in any zone Q_i is proportional to the aggregate land supply L_i^D . Then they are described as $Q_i = h_i L_i^D$ and $L_i^D = Q_i / h_i$, where coefficient h_i denotes a floor volume ratio in zone $i \in \mathbf{I}$. This approach is applicable in practice if the floor volume ratio reaches at an upper limit in any zones. In equilibrium, the zero developer's profit holds as $R_i Q_i - P_i L_i^D = (h_i R_i - P_i) L_i^D = 0$. The building rent and the land rent in any zone thus satisfy $h_i R_i = P_i$.

Aggregate Supply for Land

Aggregate supply functions for land are compared in the sixth column in Table 4.

*Exogenous land supply.*The simplest modeling of aggregate supply for land is that the supply is given as an exogenous variable to each zone $L_i^S = \bar{L}_i^S$. Only DCLD Model has employed this modeling.

Discrete choice in land supply. The logit model is employed not only to describe location choice of locators but also to formalize allocation of land to zones. DSDC Model and RURBAN Model have modeled a repetitive landowner in type $m \in \mathbf{M}$ who allocates the total amount of land $\bar{L}_m$ to zones labeled by $i \in \mathbf{I}_m$. The probability or the share of land supply to the zone L_i^S ($i \in \mathbf{I}_m$) to the total amount $\bar{L}_m$ is stated by the logit model $\exp(\gamma R_i + \ln \bar{N}_i) / \sum_{i' \in \mathbf{I}_m} \exp(\gamma R_{i'} + \ln \bar{N}_{i'})$ or $\exp(\gamma \ln R_i + \ln N_{i'}) / \sum_{i' \in \mathbf{I}_m} \exp(\ln \gamma R_i + \ln N_{i'})$, where γ is the parameter which governs the landowner's preference.

TABLE 4: Comparison of Functions

MODEL	Location attractiveness (Indirect utility or profit)	Individual demand for building space	Aggregate supply for floor	Aggregate demand for land	Aggregate supply for land
General Form	$V_{ki} = V(Y_k, R_i, e_i, E_i, \alpha_k)$	$q_{ik} = q(Y_k, R_i, e_i, E_i, \alpha_k)$	$Q_i = Q(R_i, P_i, Z_i, \beta)$	$L_i^D = L^D(R_i, P_i, Z_i, \beta)$	$L_i^S = L^S(P, W; i)$
DSDC Model	$V_{ki} = \phi(e_i, E_i, \alpha_k) - R_{ki}\bar{q}_{ki} + \ln\left(\frac{L_i}{\bar{q}_{ki}}\right)$	$q_{ik} = \bar{q}_{ik}$ $(= exogenous)$	$Q_i = h_i L_i^D$ h_i is specific to each i.	$L_i^D = Q_i / h_i$	$L_i^S = \frac{\exp(\gamma P_i + \ln \bar{N}_i)}{\sum_{i' \in \mathbf{I}_m} \exp(\gamma P_{i'} + \ln \bar{N}_{i'})} \cdot \bar{L}_m$ and $P_i = h_i R_i$ $for\ all\ i \in \mathbf{I}_m$
DCLD Model	$V_{ki} = \phi(e_i, E_i, \alpha_k) - \rho_k \ln R_i + \varphi_k \ln Y_k$ ρ_k is specific to each *k*.	$q_{ik} = \rho_k \frac{Y_k}{R_i}$	$Q_i = h_i L_i^D$ h_i is specific to each i.	$L_i^D = Q_i / h_i$	$L_i^S = \bar{L}_i^S (= exogenous)$ $for\ all\ i (= m) \in \mathbf{I}(= \mathbf{M})$
RURBAN Model	$V_{ki} = \phi(e_i, E_i, \alpha_k) - \rho_k \ln R_i + \varphi_k \ln Y_k + \ln\left(\frac{\bar{L}_i}{\bar{q}_{ki}}\right)$ ρ_k is specific to each *k*.	$q_{ik} = \rho_k \frac{Y_k}{R_i}$	$Q_i = h_i L_i^D$ h_i is specific to each i.	$L_i^D = Q_i / h_i$	$L_i^S = \frac{\exp(\gamma \ln P_i + \ln N_i)}{\sum_{i' \in \mathbf{I}_m} \exp(\ln \gamma P_{i'} + \ln N_{i'})} \cdot \bar{L}_m$ and $P_i = h_i R_i$ $for\ all\ i \in \mathbf{I}_m$
BDSB Model	$V_{ki} = \phi(e_i, E_i, \alpha_k) + \int_{R_i}^{\bar{R}} (a - bs) ds$	$q_{ik} = a - bR_i$	$Q_i = \beta_0 \beta_1 R_i^{\beta_1 - 1} P_i^{\beta_1 - 1}$	$L_i^D = \beta_0 \beta_2 R_i^{\beta_1} P_i^{-\beta_2 - 1}$	$L_i^S = \frac{(\bar{L}_i - L_i^{t-1})}{1 + \exp\{-\gamma(P_i^t - \delta P_i^{t-1}) + \varepsilon\}} + L_i^{t-1}$ $for\ all\ i (= m) \in \mathbf{I}(= \mathbf{M})$ P_i^{t-1} and L_i^{t-1} are land rent and land supply in the previous period.
CDLS Model	$V_{ki} = \phi(e_i, E_i, \alpha_k) - \rho_k R_i + \varphi_k Y_k + \ln\left(\frac{\bar{L}_i}{\bar{q}_{ki}}\right)$	$q_{ik} = \frac{\rho_k}{\varphi_k}$	$Q_i = h_i L_i^D$ h_i is specific to each i.	$L_i^D = Q_i / h_i$	$L_i^S = \frac{\exp(\gamma \ln P_i + \ln \bar{N}_i)}{\sum_{i' \in \mathbf{I}_m} \exp(\gamma \ln P_{i'} + \ln \bar{N}_{i'})} \cdot \left(1 - \frac{\lambda_m}{\left(\frac{1}{\gamma}\right) \ln \sum_{i' \in \mathbf{I}_m} \exp(\gamma \ln P_{i'} + \ln \bar{N}_{i'})}\right) \cdot \bar{L}_m$ and $P_i = h_i R_i$ $for\ all\ i \in \mathbf{I}_m$
NCUE Model Family: R-CUE	$V_{ki} = \omega_i \{\phi(e_i, E_i, \alpha_k) - \rho \ln R_i + \varphi_k \ln Y_k\}$ ω_i is the flood safety as a function of the expected water level.	$q_{ik} = \rho_k \frac{Y_k}{R_i}$	$Q_i = h_i L_i^D$ h_i is specific to each i.	$L_i^D = Q_i / h_i$	$L_i^S = \left(1 - \frac{\gamma_i}{P_i}\right) \bar{L}_i$ and $P_i = h_i R_i$ $for\ all\ i (= m) \in \mathbf{I}(= \mathbf{M})$
NCUE Model Family: G-CUE, VM-CUE	$V_{ki} = \phi(e_i, E_i, \alpha_k) - \rho_k \ln R_i + \varphi_k \ln Y_k$				

CDLS Model has also employed the logit model for allocation of land to zones but assumed that the total amount of land $L^S(\Omega_i) = \left(1 - \frac{\lambda_m}{\Omega_i}\right) \cdot \bar{L}_m$ is endogenous as a function of the expected maximum land rent in terms of the log-sum form $\Omega_i = \left(\frac{1}{\gamma}\right) \ln \sum_{i' \in \mathbf{I}_m} \exp(\gamma \ln R_{i'} + \ln \bar{N}_{i'})$.

Land supply only to the zone for each land type. Since BDSB Model and NCUE Model Family have defined a zone for each land type, the land in type $m \in \mathbf{M}$ is supplied only to the zone $i \in \mathbf{I}_m = \{m\}$. The aggregate land supply is a function of the total available land $\bar{L}_m$ and the land rent in the zone P_i^t.

Programmability

Table 5 compares programmability.

Since BDSB Model has modeled the profit of a developer, the objective function denoting the social surplus then includes it in the explicit form of $\beta_0 R_i^{\beta_1} P_i^{-\beta_2}$.

In other models, the developer's profit is a linear form as $R_i Q_i - P_i L_i^D = (h_i R_i - P_i) L_i^D$. The developer's profit in equilibrium must be zero in all zones and results in $h_i R_i = P_i$. If we consider these conditions explicitly the term $R_i Q_i - P_i L_i^D$ can be therefore deleted in the objective function and the land rent P_i can be replaced with $h_i R_i$.

Tasks for Further Development of Models

There still exist a lot of tasks for further development of the models in the family of the CUE model. We have to be engaged in the tasks.

The most critical one is the development of an efficient algorithm for searching a solution. The models are described as a large system of equations or inequalities to be solved. The computation is so tough that it takes over an hour to reach at a solution in the application to a large urban area like Tokyo. When the models are working interactively with a transport network model for traffic assignment, the computation time is furthermore critical in practical applications (*e.g.* Kim *(18)*). Although computer hardware would advance continuously, a great effort for developing the efficient algorithm for a quick searching is demanded.

Another task to be mentioned here is the development of more sophisticated techniques for parameter estimation from the view point of statistics. As described in Anselin *(37)*, spatial statistics/econometrics has been advancing both in theory and in practice so much that it may provide the CUE models with useful suggestions. Collaboration between the models in the CUE model family and spatial statistics is a way ahead to us.

5. CONCLUDING REMARKS

This paper has presented the CUE model applicable for impact analysis of urban policies including transport policies, land use regulations and infrastructure investment projects. The paper has shown a general and standard form of the CUE model. The paper has introduced several CUE models developed and applied in Japan. Each model is compared with each other in experiences of application, in mathematical function form and in programmability of equilibrium.

Since each model in the family of the CUE model is a special case of the general form, a combination of parts employed from different models can be a new member of the family. In other word, some parts of the member model can be replaced with corresponding parts in the other member model in the CUE model family. Particularly the aggregate land supply function can be used interchangeably among the models in the CUE model family.

TABLE 5: Comparison of Programmability

MODEL	Mathematical Programming
General Form	$\min_{R,P} \sum_{k\in\mathbf{K}} \frac{N_{kT}}{MIV_k} \cdot S(V_k(\cdot),\theta_k) + \sum_{i\in\mathbf{I}} \pi^D(R_i,P_i,Z_i,\beta) + \sum_{m\in\mathbf{M}} \pi^L(P_m,W_m)$
DSDC Model	$\min_{R,P} \sum_{k\in\mathbf{K}} N_{kT} \left(\frac{Y_k}{\varphi_k}\right)\left(\frac{1}{\theta_k}\right)\ln\left[\sum_{i\in\mathbf{I}_k} \exp\left\{\theta_k\left(\phi(\overline{e}_i,E_i,\alpha_k) - \rho R_i + \varphi_k \ln Y_k\right) + \ln \overline{L}_m\right\}\right]$ $+ \sum_{i\in\mathbf{I}}(R_iQ_i - P_iL_i^D) + \sum_{m\in\mathbf{M}} L_m \left(\frac{1}{\gamma}\right)\ln\left\{\sum_{i\in\mathbf{I}_m}\exp(\gamma P_i + \ln N_{kT})\right\}$ where $Q_i = h_iL_i^D$ and for $P_i = h_iR_i$ all $i \in \mathbf{I}$.
DCLD Model	$\min_{R,P} \sum_{k\in\mathbf{K}} N_{kT} \left(\frac{Y_k}{\varphi_k}\right)\left(\frac{1}{\theta_k}\right)\ln\left[\sum_{i\in\mathbf{I}_k} \exp\left\{\theta_k\left(\phi(\overline{e}_i,E_i,\alpha_k) - \rho_k \ln R_i + \varphi_k \ln Y_k\right)\right\}\right]$ $+ \sum_{i\in\mathbf{I}}(R_iQ_i - P_iL_i^D) + \sum_{m(=i)\in\mathbf{M}(=\mathbf{I})} P_mL_m$ where $Q_i = h_iL_i^D$ and for $P_i = h_iR_i$ all $i \in \mathbf{I}$.
RURBAN Model	$\min_{R,P} \sum_{k\in\mathbf{K}} N_{kT} \left(\frac{Y_k}{\varphi_k}\right)\left(\frac{1}{\theta_k}\right)\ln\left[\sum_{i\in\mathbf{I}_k} \exp\left\{\theta_k\left(\phi(\overline{e}_i,E_i,\alpha_k) - \rho \ln R_i + \varphi_k \ln Y_k\right) + \ln L_m\right\}\right]$ $+ \sum_{i\in\mathbf{I}}(R_iQ_i - P_iL_i^D) + \sum_{m\in\mathbf{M}} L_m \left\{\left(\frac{1}{\gamma}\right)\ln\sum_{i\in\mathbf{I}_m}\exp(\gamma \ln P_i + \ln N_{kT})\right\}$ where $Q_i = h_iL_i^D$ and for $P_i = h_iR_i$ all $i \in \mathbf{I}$.
BDSB Model	$\min_{R,P} \sum_{k\in\mathbf{K}} N_{kT} \left(\frac{1}{\theta_k}\right)\ln\left[\sum_{i\in\mathbf{I}_k} \exp\left\{\theta_k\left(\phi(e_i,E_i,\alpha_k) + \int_{R_i}^{R_{\max}}(a - bs)ds\right)\right\}\right]$ $+ \sum_{i=1}^{I} \beta_0 R_i^{\beta_1} P_i^{-\beta_2} + \sum_{m(=i)\in\mathbf{I}(=\mathbf{M})}\left[(\overline{L}_i - L_i^{t-1})\left(\frac{1}{\gamma}\right)\ln\left\{\exp(\gamma P_i) + \exp(\gamma P_i^{-1})\right\} + P_iL_i^{-1}\right]$
CDLS Model	$\min_{R,P} \sum_{k\in\mathbf{K}} N_{kT} \left(\frac{Y_k}{\varphi_k}\right)\left(\frac{1}{\theta_k}\right)\ln\left[\sum_{i\in\mathbf{I}_k} \exp\left\{\theta_k\left(\phi(\overline{e}_i,E_i,\alpha_k) - \rho \ln R_{ki} + \varphi_k \ln Y_k\right) + \ln \overline{L}_m\right\}\right]$ $+ \sum_{i\in\mathbf{I}}(R_iQ_i - P_iL_i^D) + \sum_{m\in\mathbf{M}} \overline{L}_m \left\{\left(\frac{1}{\gamma}\right)\ln\sum_{i\in\mathbf{I}_m}\exp(\gamma \ln P_i + \ln N_{kT}) - \lambda_m \ln\left\{\left(\frac{1}{\gamma}\right)\ln\sum_{i\in\mathbf{I}_m}\exp(\gamma \ln P_i + \ln N_{kT})\right\}\right\}$ where $Q_i = h_iL_i^D$ and for $P_i = h_iR_i$ all $i \in \mathbf{I}$.
NCUE Model Family	$\min_{R,P} \sum_{k\in\mathbf{K}} N_{kT} \left(\frac{Y_k}{\varphi_k}\right)\left(\frac{1}{\theta_k}\right)\ln\left[\sum_{i\in\mathbf{I}_k} \exp\left\{\theta_k\left(\phi(\overline{e}_i,E_i,\alpha_k) - \rho_k \ln R_i + \varphi_k \ln Y_k\right)\right\}\right]$ $+ \sum_{i\in\mathbf{I}}(R_iQ_i - P_iL_i^D) + \sum_{m(=i)\in\mathbf{M}(=\mathbf{I})} (R_m - \lambda_m \ln R_m)\overline{L}_m$ where $Q_i = h_iL_i^D$ for all $i \in \mathbf{I}$.

APPENDIX A: NOTATIONAL GLOSSAY FOR THE CUE MODEL

Variables and functions necessary for describing a general mathematical form of the CUE model are here listed. They are distinguished in the way provided by Anas and Liu *(38)*.

Labels and Sets

$i \in \{1,\cdots,I\}$: a label for a zone.

$k \in \mathbf{K} = \{1,\cdots,K\}$: a label for a locator type.

$m \in \mathbf{M} = \{1,\cdots,M\}$: a label for a landowner or a type of land.

$(i \in)\mathbf{I}_m$: the set of labels for the zones that a landowner m provides her/his land.

$\bigcup_m \mathbf{I}_m = \{1,\cdots,I\}, \quad and \quad \mathbf{I}_m \cap \mathbf{I}_{m'} = \varnothing \quad for \quad all \quad m \neq m'$.

$(i \in)\mathbf{I}_k$: the set of labels for the zones that a locator in type k can choose to locate and

$\bigcup_k \mathbf{I}_k = \{1,\cdots,I\}$.

Exogenous Variables and Vectors

$N_{ki} \in \boldsymbol{R}_+$: the number of locators in type k locating in zone i.

$N_k = [N_{k1}, \cdots, N_{kI}] \in \mathbf{R}_+^I$: the vector associated with N_{ki}.

$N_{kT} = \sum_{i \in \mathbf{I}} N_{ki} \in \mathbf{R}_+$: the total number of locators in type k.

$N = [N_1, \cdots, N_K] \in \mathbf{R}_+^{K \times I}$:a vector associated with the vector N_k.

$L_m \in \mathbf{R}_+$: the amount of land owned by a landowner in type m or the available amount of a land type m.

$E_i \in \mathbf{R}^{G'}$: the G' dimensional vector associated with the exogenous geographical/economic features or exogenous attributes of zone i.

$E = [E_1, \cdots, E_I]$: the vector associated with E_i.

Endogenous Variables and Vectors

$R_i \in \mathbf{R}_+$: the building (floor) rent in zone i.

$R = [R_1, \cdots, R_I] \in \mathbf{R}_+^I$: the vector associated with R_i.

$P_i \in \mathbf{R}_+$: the land rent in zone i.

$P = [P_1, \cdots, P_I] \in \mathbf{R}_+^I$: the vector associated with P_i.

P_m : the vector associated with the land rent in zone i which belong to the set $\mathbf{I}_m$.

$Q_i \in \mathbf{R}_+$: the aggregate building supply in zone i.

$Q = [Q_1, \cdots, Q_I] \in \mathbf{R}_+^I$: the vector associated with $Q_i \in \mathbf{R}_+$.

$L_i^D \in \mathbf{R}_+$: the aggregate land demand in zone i.

$L^D = [L_1^D, \cdots, L_I^D] \in \mathbf{R}_+^I$ the vector associated with L_i^D.

$L_i^S \in \mathbf{R}_+$: the aggregate land supply in zone i.

$L^S = [L_1^S, \cdots, L_I^S] \in \mathbf{R}_+^I$:the vector associated with L_i^S.

$e_i = e_i(N) \in \mathbf{R}^G$: the G dimensional vector associated with the endogenous geographical/economic features or endogenous attributes of zone i, which is dependent on the distribution of locators denoted by the vector N so as to indicate externality.

$e = [e_1, \cdots, e_I]$: the vector associated with e_i.

Locators

Parameters

$\alpha_k = [\alpha_{k1}, \cdots, \alpha_{kH'}]$: the H' dimensional vector associated with parameters governing the attractiveness of a zone for the locator in type k.

$\alpha = [\alpha_1, \cdots, \alpha_K]$: the vector associated with α_k.

θ_k : the parameter in the logit model for location choice of the locator k.

$\theta = [\theta_1, \cdots, \theta_K]$: the vector associated with θ_k.

Intermediate Variables

$V_{ki} = V(R_i, e_i, E_i, \alpha_k, Y_k)$: the location attractiveness (the indirect utility or profit) which a locator in type k can enjoy in i.

$V_k = [V_{k1}, \cdots, V_{kI}] \in \mathbf{R}_+^I$: the vector associated with V_{ki}.

$V = [V_1, \cdots, V_K] \in \mathbf{R}_+^{K \times I}$: the vector associated with V_k.

$q_{ik} = q(R_i, e_i, E_i, \alpha_k, Y_k)$: the individual floor apace demand of the locator k in zone i.

$a_{ki} = \dfrac{N_{ki}}{N_{kT}} = a_{ki}(V_k, \theta_k; i)$: the probability that the locator in type k chooses the zone i.

$S(V_k, \theta_k)$: the log-sum function of the locator k in the logit model for location choice.

Exogenous Variables

Y_k :the income level of the locator type k (to be ignored if the locator is not a household).

$MIV_k = MIV(R_i, e_i, E_i, \alpha_k, Y)$: the marginal utility of income of the locator k in zone i ($MIV(R_i, e_i, E_i, \alpha_k, Y) = 1$ if the locator is not a household).

Developer

Parameters

$\beta = [\beta_1, \cdots, \beta_{H''}]$:the vector associated with parameters governing a developer's technology.

Z_i : the H'' dimensional vector associated with exogenous parameters governing the aggregate building supply in zone i .

$Z = [Z_1, \cdots, Z_I]$: the vector associated with Z_i .

Intermediate Variables

$\pi_i^D = \pi^D(R_i, P_i, Z_i, \beta)$: the profit of the developer in zone i .

Landowner

Parameters

W_m : the H''' dimensional vector associated with exogenous parameters governing the aggregate land supply in zone i .

$W = [W_1, \cdots, W_M]$: the vector associated with W_m .

Intermediate Variables

$\pi_m^L = \pi^L(P_m, W_m)$: the profit of a landowner in type m .

In the above notations, $\mathbf{R}$ is a set of real number, and $\mathbf{R}_+$ is a set of non-negative real number. $\mathbf{R}^n$ denotes n dimensional Euclidean space.

APPENDIX B: REFORMULATION OF THE RUBAN MODEL

Since the RURBAN model was based simultaneously both on the random utility theory and the random bidding theory in its original formulation, the original formulation contained an inconsistency with price mechanism in market equilibrium. The RURBAN model in this paper is reformulated by the authors of this paper. The appendix explains the inconsistency in the original formulation.

The zone setting in the RURBAN model are $i = (k, m) \in \mathbf{I} = \{(k,1), \cdots, (K, M)\}$, $\mathbf{I}_k = \{(k,1), \cdots, (k, M)\}$ and $\mathbf{I}_m = \{(1,m), \cdots, (K,m)\}$. For simplicity, we here replace ki with $(k,m) = km$ and we assume that the geographical/economic features of the zone $i = (k,m) = km$ can be specified $e_i = e_{(k,m)} = e_m$ and $E_i = E_{(k,m)} = E_m$ for all i .

In tradition of urban economics, the utility maximization of a household is formalized as,

$$V_{km} = V(R_{km}, e_m, E_m, \alpha_k, Y_k) = \max_{z_{km}, q_{km}} u(z_{km}, q_{km}, e_m, E_m, \alpha_k, Y_k) \quad \text{(A1)}$$

$$s.t. \quad z_{km} + R_{km} q_{km} = Y_{km} \, . \quad \text{(A2)}$$

where $V(\cdot)$ is the indirect utility function, $u(\cdot)$ is the direct utility function and z is the consumption of the composite good. The maximization yields to the individual demand function for building space (land in the original RURBAN model) as

$$q_{km} = q^V(R_{km}, e_m, E_m, \alpha_k, Y_k) \, . \quad \text{(A3)}$$

Bid rent function in urban economics is derived from the following maximizing problem.

$$B_{km} = B(V_{km}, e_m, E_m, \alpha_k, Y_k) = \max_{z_{km}, q_{km}} (Y_k - z_{km}) / q_{km} \, , \quad \text{(A4)}$$

$$s.t. \quad u(z_{km}, q_{km}, e_m, E_m, \alpha_k, Y_k) = V_{km} \, . \quad \text{(A5)}$$

The maximization of the rent also yields to the individual demand for building.

$$q_{km} = q^B(V_{km}, e_m, E_m, \alpha_k, Y_k) \, . \quad \text{(A6)}$$

Since the bid rent maximization stated by (A4) and (A5) is mutually consistent with the utility maximization by (A1) and (A2), the following conditions must hold.

$$q^B(V_{km}(R_{km},e_m,E_m,\alpha_k,Y_k),e_m,E_m,\alpha_k,Y_k) = q^V(R_{km},e_m,E_m,\alpha_k,Y_k)\,, \tag{A7}$$

and $$q^V(B_{km}(V_{km},e_m,E_m,\alpha_k,Y_k),e_m,E_m,\alpha_k,Y_k) = q^B(V_{km},e_m,E_m,\alpha_k,Y_k)\,. \tag{A8}$$

In an equilibrium state, the original RURBAN-model defined the building (land) rent in the equilibrium state which appears in the indirect utility function as,

$$R_m^* = \left(\frac{1}{\gamma}\right)\ln\left\{\sum_{k\in\mathbf{K}}\exp(\gamma B_{km})\right\} \in \mathbf{R}_+\,. \tag{A9}$$

On the other hand, the indirect utility appears in the bid rent function is also defined in the logsum form as.

$$V_k^* = \left(\frac{1}{\theta_k}\right)\ln\left\{\sum_{m\in\mathbf{M}}\exp(\theta_k V_{km})\right\} \in \mathbf{R}\,. \tag{A10}$$

In (A9) and (A10), * denotes the equilibrium. The original RURBAN model however defined the indirect utility and the bid rent functions as,

$$V_{km} = V(R_m^*,e_m,E_m,\alpha_k,Y_k) \neq V(R_{km},e_m,E_m,\alpha_k,Y_k)\,, \tag{A11}$$

and $$B_{km} = B(V_k^*,e_m,E_m,\alpha_k,Y_k) \neq B(V_{km},e_m,E_m,\alpha_k,Y_k)\,. \tag{A12}$$

Considering (A7) and (A8) with (A11) and (A12) results in

$$q^B(V_k^*,e_m,E_m,\alpha_k,Y_k) \neq q^V(R_m^*,e_m,E_m,\alpha_k,Y_k)\,. \tag{A13}$$

The original RURBAN model therefore was not successful in consistently formalizing the individual demand for building space since (A7) and (A8) have been violated.

Restrictions on parameters in random biding model and random utility model stated in (A9) and (A10) respectively were proposed in the original RURBAN model. By notations in this paper, the restrictions seem to be $\theta_k\rho_k = \gamma$ for all $k \in \mathbf{K}$ and $m \in \mathbf{M}$, where ρ_k is the parameter associated with bid rent. The restrictions on these parameters are bridging the locator's behavior and the landowner's one in an ad-hoc way. Locators (demand side) and landowners (supply side) behave independently but interactively through price information in the land markets. The restrictions on the parameters thus violated a fundamental principle of market equilibrium model. The original RURBAN model was not able to assess the sharing of the benefit of an urban policy explained in this paper.

The above inconsistency is the reason why this paper has reformulated the RURBAN model.

REFERENCES

(1)Alonso, W. *Location and Land Use*. Cambridge, Mass., Harvard University Press, 1964.

(2)Anas, A. Residential Location Markets and Urban Transportation. Academic Press, 1982.

(3)Anas, A. *Modeling in Urban and Regional Economics*. Fundamentals of Pure and Applied Economics 26, harwood academic publishers, 1987.

(4)Shoven, J.B., and Whalley, J. Applied General Equilibrium Models of Taxation and International Trade : An Introduction and Survey. *Journal of Economic Literature*, Vol.22, 1984, pp.1007―1051.

(5)Mills, E.S. *Urban Economics*. Scott, Foresman & Co., Glenview, III, 1970.

(6)Vickerly, W. Pricing as a Tool in the Coordination of Local Transportation, In: *Transportation Economics*, National Bureau of Economic Research, New York, 1965.

(7)Fujita, M. *Urban Economic Theory*. Cambridge University Press, Cambridge, 1989.

(8)Muth, R.F. *Cities and Housing*, Chicago, University of Chicago Press. 1969.

(9)Mills, E.S. An Aggregative Model of Resource Allocation in a Metropolitan Area, *American Economic Review*, Vol.57, 1967, pp.197―210.

(10)Herbert, J.S., and B.H. Stevens. A Model of the Distribution of Residential Activity in Urban Areas, *Journal of Regional Science*, Vol.2. 1960, pp.21―36.

(11)Foot, D. Operational Urban Models : An Introduction. Methuen, 1981.

(12)Lowry, I.S. *A Model of Metropolis*. RM-4035-RC, the Rand Corporation, Santa Monica CA. 1964.

(13)Webster, F. V., P. H. Bly and N. J. Paulley, editors, Urban Land-use and Transport Interaction, Policies and Models, Avebury (Gower Publishing Company Ltd.), 1988.

(14)Wegener, M. Overview of Land-Use Transport Models. *Proceedings of CUPUM '03 Sendai* (Conference Brochure of CUPUM '03 Sendai, pp.20—40), The 8th International Conference on Computers in Urban Planning and Urban Management, 2003.

(15)Vairan . H.R. *Microeconomic Analysis*. Norton, 1992.

(16)Miyagi, T. On the formulation of a stochastic user equilibrium model consistent with the random utility theory ─A Conjugate dual approach, *Proceedings of the 4th World Conference on Transport Research*, Vol.2, 1986, pp.1619─1635.

(17)Oppenheim, N. Urban Travel Demand Modeling : From Individual Choices to general Equilibrium. John Wiley & Sons, New York, N.Y. 1995.

(18)Kim, T. J. Integrated Urban Systems Modeling: Theory and Applications, Kluwer Academic Publihers, 1989.

(19)Negishi,T, Welfare economics and the existence of an equilibrium for a competitive economy, *Metroeconomica*, 12, pp.92─97, 1960

(20)Hayashi, Y., and K. Doi. A Model for Analysing the Imputation of Commuters' Benefits to Land Property Values. In: *Selected Proceedings of the 5th World Conference on Transport Research*, 1989, pp.303─317.

(21)Hayashi, Y., and Y. Tomita. A micro-analytic residential mobility model for assessing the effects of transport improvement. In: *Selected Proceedings of the 5th World Conference on Transport Research*, 1989, pp.91—105.

(22)Morisugi, H., and E. Ohno. A benefit incidence matrix for urban transport improvement. *Papers in Regional Science : the Journal of the RSAI*, Vol.71, No.1, 1992, pp.53—70.

(23)Morisugi, H., E. Ohno, and T. Miyagi. Benefit Incidence of Urban Ring Road –Theory and Case Study of the Gifu Ring Road. *Transportation*, Vol. 20, 1993, pp.285─303.

(24)Miyamoto, K., and K. Kitazume. A land use model based on random utility/rent-bidding analysis (Rurban). In: *Selected Proceedings of the 5th World Conference on Transport Research*, Vol.4, 1990, pp.107─121.

(25)Miyamoto, K., T. Noami, Y. Kuwata, and K. Yokozawa. An evaluation method of transport projects with the aid of RURBAN Model. In: *Selected Proceedings of the 6th World Conference on Transport Research*, Vol.1, 1993, pp.55─66.

(26)Miyamoto, K., V. Vichiensan, N. Sugiki, and K. Kitazume. Applications of RURBAN Integrated with a Transport Model in Detailed Zone System. In: *Proceedings of the 11th World Conference on Transport Research*, CD-ROM, 2007.

(27)Ueda, T., K. Hiratani, and M. Tsutsumi. Landuse model based on the general equilibrium of land and building markets. In: *Proceedings of International Conference on Land Problem and Urban Policy*, Kyoto, 1993, pp.183─198.

(28)Ueda, T., M. Tsutsumi, and H. Nakamura. An Urban Transport and Activity Location Model for the Evaluation of Commuting Rail Improvement, *presented at the 7th World Conference of Transportation Research, Sydney, Australia, 16-21 July, 1995.* (Available: http://surveyor.sk.tsukuba.ac.jp/paper/7th_wctr.pdf)

(29)Yoon, J.J., Y. Aoyama, D. Nakagawa, and R. Matsunaka. A Practical Land Use-Transport Interaction Model Considering Changes in Population and Industrial Activities. *Proceedings of International Symposium on City Planning 2000 Kobe*, Japan, 2000, pp.381─390.

(30)Yoon, J.J., Y. Aoyama, D. Nakagawa, and R. Matsunaka. An Evaluation of Urban and Transport Policy Using a Land-Use/Transport Interaction Model. In: *Selected Proceedings of the 9th World Conference on Transport Research*, CD-ROM, No.6114, 2003.

(31)Takagi, A., and T. Ueda. Evaluation of Flood Mitigation Countermeasures Considering the Interdependence Between Flood Risk and Land Use. *First Annual IIASA-DPRI Meeting*, 2001. (Available: http://www.iiasa.ac.at/Research/RMS/dpri2001/paper_pdf/papers.html)

(32)Takagi, A., S. Muto, and T. Ueda. The Benefit Evaluation of Urban Transportation Improvements with Computable Urban Economic Model. *'99 Shanghai International Symposium on Urban Transportation Proceedings*, 1999, pp.87—99.

(33)Muto, S., A. Takagi, and T. Ueda. The Benefit Evaluation of Transport Network Improvement with Computable Urban Economic Model. In: *Selected Proceedings of the 9th World Conference on Transport Research*, Elsevier Science, CD-ROM, No.6218, 2003.

(34)Muto, S., T. Ueda, K. Yamaguchi, and K. Yamasaki. Evaluation of Environmental Pollutions Occurred by Transport Infrastructure Project at Tokyo Metropolitan Area, In: *Selected Proceedings of the 10th World Conference on Transport Research*, CD-ROM, No.1152, 2004.

(35)Yamasaki, K., T. Ueda, and S. Muto. The Evaluation of the Tokyo-Metropolitan Area Policy by Computable Urban Economic Model (CUE). In: *Selected Proceedings of the 11th World Conference on Transport Research*, University of California, Berkeley, CD-ROM, No.734, 2007.

(36)Yamasaki, K., T. Ueda, and S. Muto. Impacts of Transport Infrastructure Policies in Population-Declining Metropolitan Area —Business Productivity and Quality of Urban Life in Tokyo—. *11th Uddevalla Symposium 2008*, Kyoto, Japan, 2008.

(37)Anselin, L. *Spatial Econometrics*. Kluwer Academic, 1988.

(38)Anas, A. and Y. Liu. A Regional Economy, Land Use, and Transportation Model (REULU-TRAN): Formulation, Algorithm Design, and Testing, *Journal of Regional Science,*Vol.47, pp.415–455, 2007.