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Spontaneous institutions as cooperative equilibrium in repeatedly-played and linked games

Takayuki Ueda	Yukihiro Ogawa	Tomoki Ishikura
Dept. of Civil Engineering	Dept. of Civil Engineering	Dept. of Civil Engineering
University of Tokyo	University of Tokyo	University of Tokyo
email:tueda@civil.t.u-tokyo.ac.jp	email:	email:

Abstract

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Keywords

Note:

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Takayuki UEDA,

Yukihiro OGAWA

and Tomoki ISHIKURA

Department of Civil Engineering, The University of Tokyo

Hongo 7-3-1, Bunkyo-ku, Tokyo 113-8656, JAPAN

E-mail tueda@civil.t.u-tokyo.ac.jp

Abstract:

Some types of social capital are revealed as a spontaneous institution in a society. The spontaneous institution has been formed and maintained through the process in which each individual keeps an incentive to behave cooperatively. This paper shows a theoretical framework and numeral examples for the spontaneous institution in this line of thought. The concept of repeated games has already provided the theoretical framework that enables us to describe the spontaneous institution as cooperative equilibrium. The recent theory of Comparative Institutional Analysis, proposed by Aoki (2001), has added an idea of linked games to the tradition of repeated games. In the model of linked games, the cooperative equilibrium is more sustainable due to a trigger strategy. Suppose there are two activities, for instance, mutual help at a disaster situation as one activity and a community festival as another acitvitiy.. The linkage of these games generates a pay-off matrix for combinations of strategies in each activity. The cooperative behavior played simultaneously in both activities may become more sustainable in repeatedly-played stages than that in played in a single activity game. The model developed by Aoki has, however, still shown the very limited cases and only considered the simple TIT FOR TAT. This paper enlarges the idea of linkage of activities and individual strategies so as to describe the vitality and robustness of spontaneous institutions

1 Introduction

Social capital like trust and tie among people in a community are a foundation for activities in which each individual is helping with other. We can confirm this in fact when we observe activities like crime-prevention for community security or town festival, school supporting, environmental preservation, and so on

An activity in a community has been repeated and sustained as a spontaneous institution in many cases. Vitality and robustness of the institution, however quietly differ among communities. A instructive general theory for understanding such differences has not been developed yet.

This paper aims at proposing a game theoretic approach for understanding a mechanism in which a spontaneous institution is formed and sustained as a cooperative equilibrium. We model the game which links some activities in a community and is repeatedly played in infinite time horizon. The paper then illustrates a variety of spontaneous institutions attained as a cooperative equilibrium in the game by numerical analysis. We interpret each of the numerically-illustrated cooperative equilibrium in a variety of social or economic contexts.

This paper is organized as follows. Section 2 first reviews mathematical models related to our special interests, then formulates a baseline model for our analysis and finally rewrites it to a simplified game in the case of two activities and two persons. Section 3 analyzes a variety of equilibrium solutions in the simplified game by numerical calculation. Section 4 concludes the paper and refers to some directions in further research.

2 Model

2.1 Review of models related to our special interests

Mathematical sociology has proposed many models describing trust and tie in a community as a social network. Wasserman and Faust (1994) have instructively overviewed a variety of social network models and the related techniques for applying them to sociologically-interesting topics.

Social network modeling has advanced in theories of complex network in some directions that emphasize mathematical features of network configuration and dynamic processes of network formation. Watts (1999) has illustrated a filed of complex network and interpreted the networks in sociological contexts.

Advanced game theories have been more interested in non-cooperative game than cooperative one. Fudenberg and Tirole(1996) has shown that a main stream of game theory focuses on non-cooperative game, while Moilin (1995) has impressed the applicability of cooperative game to a wide rage of social or economic issues.

Game theoretic ideas and complex network modeling have been combined so that they can turn out fruitful economic interpretations and implications. Demange and Wooders (2005), has succeeded in modeling group formation for networks, clubs, and coalitions, Vega-Redondo (2007) has also succeeded in modeling social networks. Goyal (2007) has clearly illustrated how a game theoretic model can be reformulated in a network.

On the other hand, the recent theory of Comparative Institutional Analysis, proposed by Aoki (2001), has added an idea of linked games to the tradition of repeated games. In the model of linked games, the cooperative equilibrium is more sustainable due to TIT FOR TAT strategy. Suppose there are two domains, for instance, mutual help at a disaster situation as one domain and that at a community festival as another domain.

The linkage of these games generates a pay-off matrix for combinations of strategies in each domain. The cooperative behavior played simultaneously in both domains may become more sustainable in repeatedly-played stages than that in played in a single domain. The model developed by Aoki has, however, still shown the very limited case of the linkage between domain games different in their domains and only considered the simple TIT FOR TAT.

The model that we are to build is based on the model by Aoki(2001) rather than theories on games in network and those on complex networks. Although a representation of social network is important and interesting in social capital study, the idea of a linked game is so complicated that it could not be installed to those network-oriented theories. In addition, the idea of a repeated game focuses on the equilibriums in a very long time horizon and often in an infinite time horizon. The evolving process of network described in the network-oriented theories would be less interesting if we focus on equilibriums in such long time span. This paper enlarges the idea of linkage of activities and individual strategies so as to describe the vitality and robustness of spontaneous institutions

.2.2 A Baseline Model

Community

There is a community where individuals, labeled by $i \in \mathbf{I} = \{1, \cdots, I\}$, are residing. There potentially exist activities in the community, each of which is labeled by $j \in \mathbf{J} = \{1, \cdots, J\}$.

Activity Game

Each activity $j \in \mathbf{J}$ itself is a game. This correspond to a domain in Aoki(2001). We call this an activity game and let it be denoted by G^j. An individual in a community is a player of the activity game. A strategy of the player $i \in \mathbf{I}$ is denoted by $\{s_i\}_{i\in\mathbf{I}}$, and a payoff of the player is by $\{V_i\}_{i\in I}$. Then the activity game is stated as

$$G^j = \left[\mathbf{I}, \left\{s_i^j\right\}_{i\in\mathbf{I}}, \left\{V_i^j\right\}_{i\in\mathbf{I}}\right]. \tag{1}$$

A player can make a choice between cooperate or not cooperate in the activity game. "Cooperate" means that the player join the activity and "not cooperate" does that she doesn't join. The strategy is denoted by

$$s_i \in \{1 \text{ (cooperate)}, 0 \text{ (not cooperate)}\} \text{ for all } i \in \mathbf{I}. \tag{2}$$

A payoff of "cooperate" includes not only the share of outcomes by cooperative action in the activity but also enjoyment from mental tie with other individuals in cooperation, mutual trust and confidence.

Any player cooperating in an activity spends g^j as an equal contribution out of her initial endowment of resource ω_i. An outcome of the activity $j \in \mathbf{J}$ denoted by y^j is an output with an input of total contribution. We can write the total contribution as

$$\sum_{i\in I} s_i^j g^j, \tag{3}$$

The outcome y^j is a function of the total contribution as

$$y^j = \left(\sum_{i\in I} s_i^j g^j\right)^{f_j}, \tag{4}$$

and is regarded as a kind of club goods.

Linked Game

Each activity in a community is linked with other. Hence we can state a linked game which contains strategies and payoffs in all activity games as,

$$G = \underset{j\in\mathbf{J}}{\times} G^j = \left[\mathbf{I}, \underset{j\in\mathbf{J}}{\times}\left\{s\right\}_{i\in\mathbf{I}}^{j}, \underset{j\in\mathbf{J}}{\times}\left\{V_i^j\right\}_{i\in\mathbf{I}}^{j}\right]. \tag{5}$$

A linkage of two activities generates "plus sum " benefit for a player cooperating in both activities. Mutual tie, trust and confidence between players formed in one activity may be positively feed-backed to those in another activity. This is the most important aspect of a linked game that we model. We call this additional payoff by linkage and let it be denoted by ΔV_i^j. This is formalized as

$$\Delta V_i^j = \sum_{\substack{j,j'\in\mathbf{J}\\(j\neq j')}} s_i^{j'}\left(\sum_{i'(\neq i)\in\mathbf{I}} s_{i'}^{j}\alpha^{jj'}y^{j'}\right), \tag{6}$$

where $\alpha^{jj'}, 0 \le \alpha^{jj'} \le 1$, is the linkage factor between activity games G^j and $G^{j'}$.

A payoff of the player $i \in \mathbf{I}$ in a linked game G is written as,

$$V_i = \left(\omega_i - \sum_{j\in\mathbf{J}} s_i^j g^j\right)^e + \left\{\sum_{j\in\mathbf{J}} s_i^j\left[y^j + \sum_{\substack{j,j'\in\mathbf{J}\\(j\neq j')}} s_i^{j'}\alpha^{jj'}y^{j'}\right]\right\}^{e^J}, \tag{7}$$

where e and e^J are a positive parameter for the utility of private good and for that of outcome by activities in cooperation respectively.

Infinitely Repeated Game

A linked game stated above is played repeatedly in infinite time horizon. The liked game is regarded as a component game in the infinitely repeated game. Responses to each player's strategy are called a trigger in common knowledge among all players. We employ the following three types of triggers.

Trigger A : activity with indispensable requirement for cooperation

If a player $i \in \mathbf{I}$ does not cooperate in a particular activity, then others do not cooperate in any activities in the next period. The particular activity is called the activity with indispensable requirement for cooperation.

Trigger B : minimum number of activities with requirement for cooperation

If a player $i \in \mathbf{I}$ cooperates in less than or equal to a number of activities, for example n, then others do not cooperate in any activities in the next period. The minimum number of activities with requirement for cooperation may play a role of index for tolerance in a community.

Trigger C TIT FOR TAT

If a player $i \in \mathbf{I}$ takes a strategy, then others imitate it or take the same strategy in the next period.

Equilibrium

We seek for the Nash equilibrium stated by the Folk Theorem (See Fundenberg and Tirole(1996)). The discount factor $\delta = 1 / (1 + \rho)$ must be installed to evaluate the payoff in terms of present value.

2.2 A Simplified Game -Two Activity/Two Person Game-

We simplify the baseline model formalized in the previous subsection so that the simplified mode can be used in numerical simulation in the next section.

In a simplified game, there exist two activities $j, j' \in \mathbf{J}$ and two individuals $i, i' \in \mathbf{I}$ in a community. A payoff of a player is represented by that of the player i as,

$$V_i = \left(\omega_i - s_i^j g^j - s_i^{j'} g^{j'}\right)^e + \left\{ s_i^j \left(y^j + s_i^{j'} s_{i'}^j \alpha^{jj'} y^{j'}\right) + s_i^{j'} \left(y^{j'} + s_i^j s_{i'}^{j'} \alpha^{jj'} y^j\right) \right\}^{e^J} \tag{8}$$

$$\text{and} \quad y^j = (s_i^j g^j + s_{i'}^j g^j)^{f^j} \text{ and } y^{j'} = (s_i^{j'} g^{j'} + s_{i'}^{j'} g^{j'})^{f^{j'}}. \tag{9}$$

Triggers from A to C are formalized as follows.

Trigger A $\quad s_i^j(t+1) = s_i^j(t) s_{i'}^{j'}(t)$ and $s_i^{j'}(t+1) = s_i^{j'}(t) s_{i'}^{j'}(t)$ (11)

where we assume that $G^{j'}$ is the activity with indispensable requirement for cooperation..

Trigger B

$$s_a^j(t+1) = s_i^j(t) \left\{ 1 - \prod_{m \in \mathbf{J}^n} \left(1 - s_{i'}^m(t)\right) \right\} \tag{12}$$

where the minimum number of activities with requirement for cooperation is dented by n. In numerical analysis we examine the cases for $n = 1$ and $n = 2$. Then we use the notation Trigger B1 and Trigger B2 for them respectively.

Trigger C

$$s_i^j(t+1) = s_{i'}^j(t) \tag{13}$$

3 Numerical Analysis

3.1 Notation for equilibrium

This section describes equilibriums in a various setting of parameters in the simplified game that we have stated in the previous section. In order to indicate a type of equilibrium simply, we employ the following notation.

$$(x, y) \times (z, w) = (s_i^j, s_i^{j'}) \times (s_{i'}^j, s_{i'}^{j'}). \tag{14}$$

3.2 Numerical Results

Linkage Factor and Discount Factor

We first examine the effects of linkage factor α and discount factor δ on the equilibrium in the infinitely repeated game. Figures numbered from 1 to 4 illustrate results on the parameter setting of ω=120, $g^j = g^{j'}$=10, $f^j = f^{j'}$=1, e=0.8, $e^j = e^{j'}$=0.4, 0<$\alpha_{jj'}$<1, 0<δ<1.

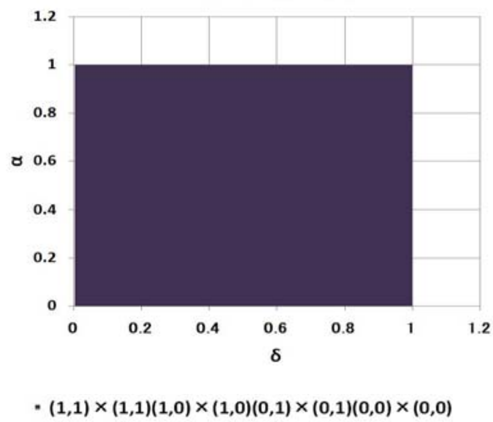


Figure 1 Trigger A in a variety of combinations of δ and α

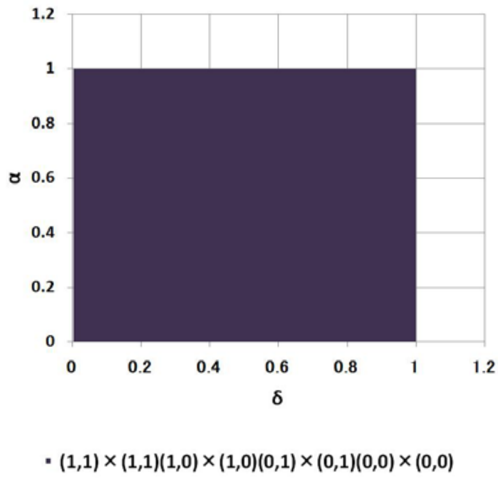


Figure 2 Trigger B1 in a variety of combinations of δ and α

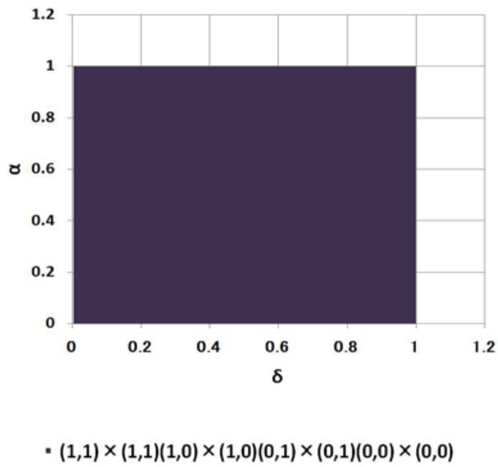
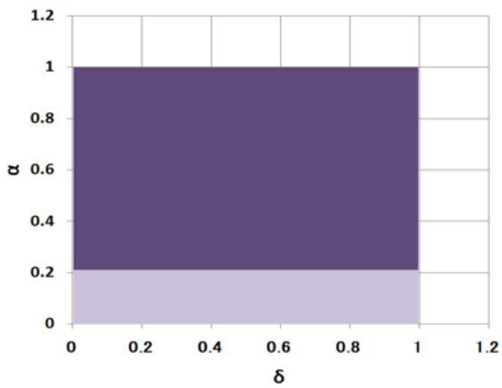


Figure 3 Trigger B2 in a variety of combinations of δ and α



▪ (1,0) × (1,0)(0,1) × (0,1)　　▪ (1,1) × (1,1)(1,0) × (1,0)(0,1) × (0,1)

Figure 4　Trigger C in a variety of combinations of δ and α

Although we had expected that a distribution of equilibrium differs so much among triggers, we have found that the almost same distribution is attained among Trigger A, B1 and B2. The combination of linkage factor α and discount factor δ is indicated by a point in a panel in the figures. A violet point indicates that all types of equilibrium *(1,0)×(1,0), (1,0)×(1,0)), (0,1)×(0,1)),(0,0)×(0,0) are attainable with the given* combination of linkage factor α and discount factor δ. Among Trigger A, B1 and B2, the linkage factor α is not always dominant for attaining the cooperation in the community.

In the case of Trigger C, as shown in Figure 4, we have found that there exist two colored areas. In both areas, the equilibrium *(0,0)×(0,0)* cannot be attained.. Trigger C eliminates the state that any individual cooperates in neither activities. Compared with other types of triggers, Trigger C (TIT FOR TAT) may lead more likely to cooperation in activities.

Contribution

We then examine effects of individual contribution for each activity, $g^j, g^{j'}$. Figures numbered from 5 to 8 illustrate results on the parameter setting of ω *=120, δ=0.8, α=0.8,* $f^j = f^{j'} = 1$, $e = 0.8$, $e^j = e^{j'} = 0.4$, $0 < g^j < 120$, $0 < g^{j'} < 120$.

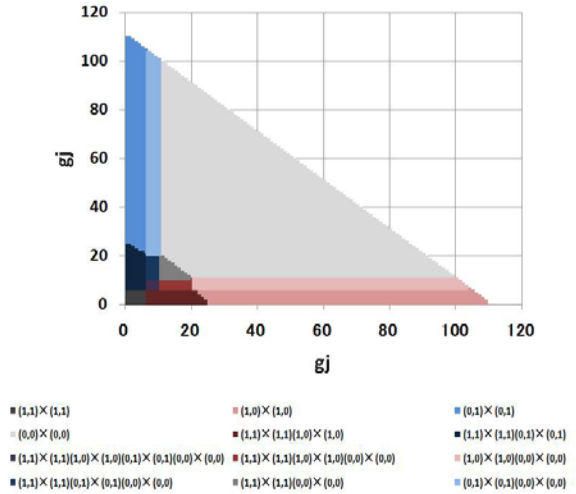


Figure 5　Trigger A　in a variety of combinations of g^{j} and $g^{j'}$

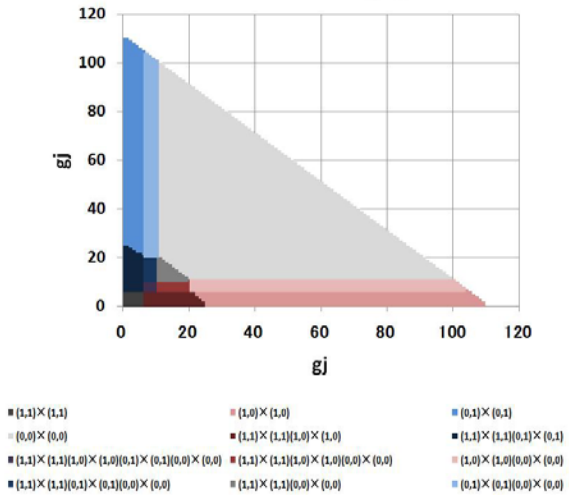


Figure 6　Trigger B1　in a variety of combinations of g^j and $g^{j'}$

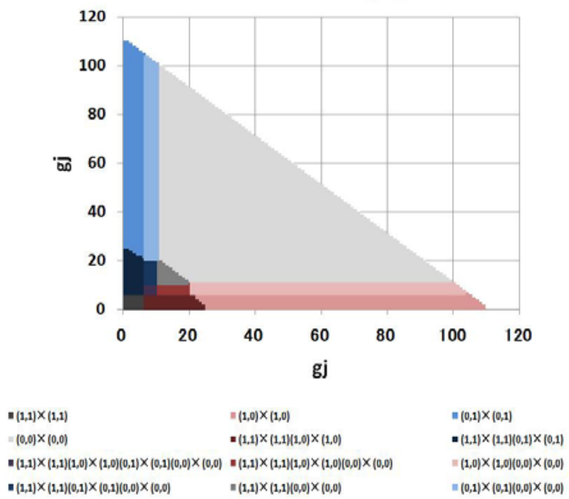


Figure 7　Trigger B2　in a variety of combinations of g^j and $g^{j'}$

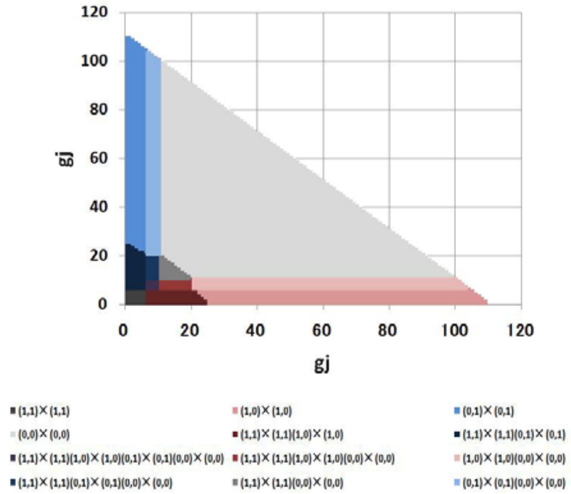


Figure 8 Trigger C in a variety of combinations of g^j and $g^{j'}$

In any cases, if the required contribution is low in both activities, then both players cooperate in both activities. The area including only *(1,1)×(1,1)* is indicating the conditions for all individuals to cooperate all activities.

We have found the area including only *(1,0)×(1,0)* and the other area including only *(0,1)×(0,1)*. These areas indicate that an individual may cooperates in one activity which requires the lower contribution than the other activity does. This is common among any triggers.

Income Level

An initial endowment of resource for each individual, denoted by ω_i or ω_i, is regarded as a income. Income distribution among individuals may govern the attainability of cooperative equilibrium. Figures numbered from 9 to 12 illustrate results on the parameter setting of δ=0.8, α=0.8, $g^j = g^{j'}$=10, $f^j = f^{j'}$=1, e =0.8, $e^j = e^{j'}$=0.4 and 0<ω<120.

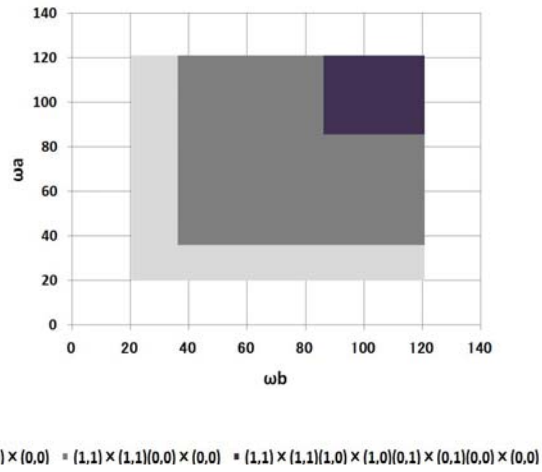


Figure 9 Trigger A in a variety of combinations of ω_i and $\omega_{i'}$

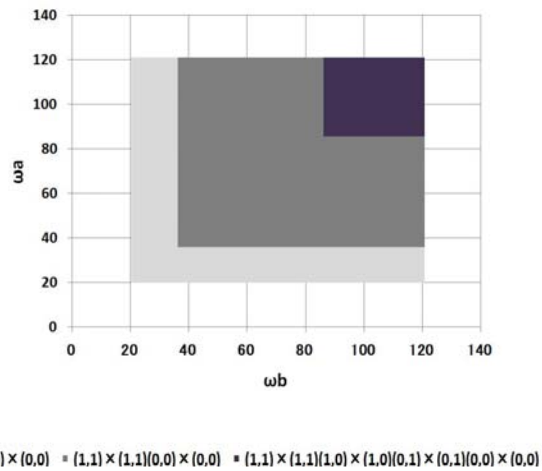


Figure 10 Trigger B1 in a variety of combinations of ω_i and $\omega_{i'}$

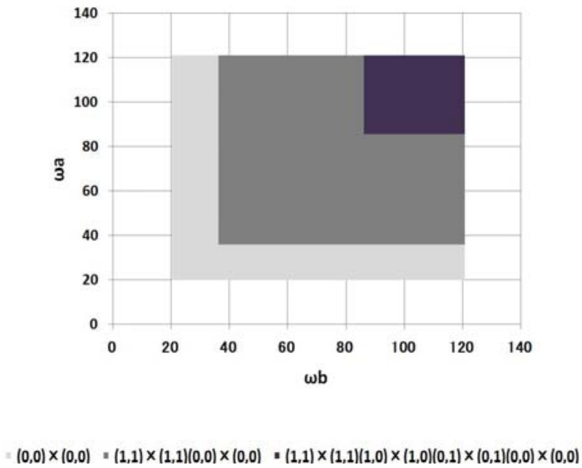


Figure 11 Trigger B2 in a variety of combinations of ω_i **and** $\omega_{i'}$

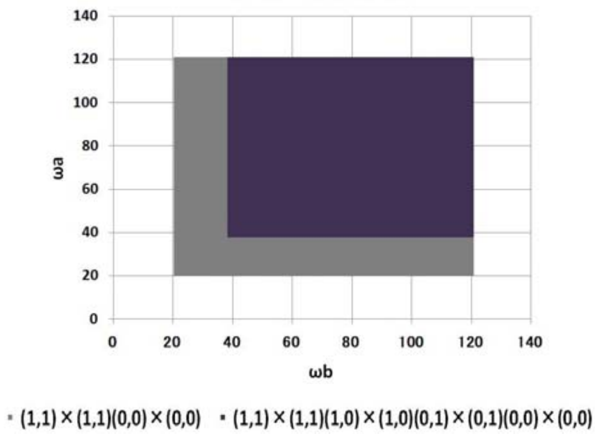


Figure 12 Trigger C in a variety of combinations of ω_i **and** $\omega_{i'}$

In cases of Trigger A, B1 and B2, we have found that an area including only *(0,0)×(0,0)* has emerged. The area is at a location where one of two individuals is rich (high income) while another is poor (low income) or where both individuals are poor.

As income level of either of individual increases, an area including cooperative equilibrium *(1,1)×(1,1)* has emerged. As both individuals become rich, an area including all types of equilibrium *(1,1)×(1,1),(1,0)×(1,0),(0,1)×(0,1) and (0,0)×(0,0)* has appeared. This implies that a community of rich people may have a variety of cooperative activities.

In case of Trigger C(TIT FOR TAT), as shown in Figure 12, we have not found an area including only $(0,0)\times(0,0)$. Compared with other triggers, Trigger C may lead more likely to cooperation in activities even when one of two individuals is poor or when both are poor.

Comparison between two communities

Vitality and robustness of a cooperative activity differs among communities. We here compare two communities, one of which is an individualistic community and the other is a group-minded community. We name the first community I-Community and the second one G-community. It is often the fact that G-Community has a higher level of vitality and robustness of a cooperative activity than I-Community does.

Numerical results in a variety of income distributions shown above have already suggested that a community where all individuals are poor can attain no cooperation and also that a community where all individuals are rich may succeed in realizing cooperation in some or all activities. However, in general, a community in country-side tends to show high vitality and robustness of a cooperative activity even though income level in the community is very low. On the other hand, a community in urban area often shows poor cooperation although income level is very high.

We numerically illustrate that an I-Community of high income people is very fragile in sustaining cooperative activities as a spontaneous institution, and also that a G-community of low income people is so tough in sustaining them.

Suppose that there exists an I-Community in urban area, We assume that the

I-Community is a group of high-income individualists in which linkage of activities does not increase any individual's payoff. We also assume that since each individual is not interested in long-term relationship with other, the individual has a low discount factor. With these assumptions, we can employ the setting of parameters $\omega=210$, $\delta=0.4$, $\alpha=0$, $g^{j}=g^{j'}=10$, $f^{j}=f^{j'}=1$, $e=0.8$. $e^{j}=e^{j'}=0.4$. The value of $\alpha=0$ here is crucial. All individuals are to take a strategy of Trigger B1. This is in common knowledge for the infinitely repeated game in the community.

In the same manner as the above, suppose that there exists a G-Community in country-side. We assume low income, high linkage between activities, and high discount factor. Then we can employ the setting of parameters□ $\omega=60$, $\delta=0.8$, $\alpha=1$, $g^{j}=g^{j'}=10$, $f^{j}=f^{j'}=1$, $e=0.8$, $e^{j}=e^{j'}=0.4$ and adopt Trigger A.

Equilibrium and payoff in each of I-Community and G-Community are indicated in Table 1 and 2 respectively.

Table 1 Equilibrium in I-Community

i \ *i'*	(1, 1)	(1, 0)	(0, 1)	(0, 0)
(1, 1)	V_i^∞=121.93	V_i^∞=120.59	V_i^∞=120.59	V_i^∞=119.60
	$V_{i'}^\infty$=121.93	$V_{i'}^\infty$=121.05	$V_{i'}^\infty$=121.05	$V_{i'}^\infty$=120.12
(1, 0)	V_i^∞=121.05	V_i^∞=121.05	V_i^∞=119,71	V_i^∞=119.88
	$V_{i'}^\infty$=120.59	$V_{i'}^\infty$=121.05	$V_{i'}^\infty$=119.71	$V_{i'}^\infty$=120.12
(0, 1)	V_i^∞=121.05	V_i^∞=119.71	V_i^∞=121.05	V_i^∞=119.88
	$V_{i'}^\infty$=120.59	$V_{i'}^\infty$=119.71	$V_{i'}^\infty$=121.05	$V_{i'}^\infty$=120.12
(0, 0)	V_i^∞=120.12	V_i^∞=120.12	V_i^∞=120.12	V_i^∞=120.12
	$V_{i'}^\infty$=119.60	$V_{i'}^\infty$=119.88	$V_{i'}^\infty$=119.88	$V_{i'}^\infty$=120.12

Table 2 Equilibrium in G-Community

i \ *i'*	(1, 1)	(1, 0)	(0, 1)	(0, 0)
(1, 1)	V_i^∞=141.48	V_i^∞=133.78	V_i^∞=126.31	V_i^∞=129.97
	$V_{i'}^\infty$=141.48	$V_{i'}^\infty$=131.14	$V_{i'}^\infty$=132.45	$V_{i'}^\infty$=132.28
(1, 0)	V_i^∞=131.14	V_i^∞=132.01	V_i^∞=130.34	V_i^∞=131.20
	$V_{i'}^\infty$=133.78	$V_{i'}^\infty$=132.01	$V_{i'}^\infty$=131.20	$V_{i'}^\infty$=132.28
(0, 1)	V_i^∞=132.45	V_i^∞=131.20	V_i^∞=132.45	V_i^∞=131.20
	$V_{i'}^\infty$=126.31	$V_{i'}^\infty$=130.34	$V_{i'}^\infty$=132.45	$V_{i'}^\infty$=132.28
(0, 0)	V_i^∞=132.28	V_i^∞=132.28	V_i^∞=132.28	V_i^∞=132.28
	$V_{i'}^\infty$=129.97	$V_{i'}^\infty$=131.20	$V_{i'}^\infty$=131.20	$V_{i'}^\infty$=132.28

Table 1 has shown that the I-Community can attain all types of equilibrium as the Nash equilibrium *(1,1)×(1,1),(1,0)×(1,0),(0,1)×(0,1) and (0,0)×(0,0)*, while Table 2 has shown that the G-community can attain only *(1,1)×(1,1) and (0,1)×(0,1)*.

The G-Community may succeed in realizing cooperation in both activity or in the activity with indispensable requirement for cooperation. The payoff in the equilibrium *(1,1)*×*(1,1)* is higher than that in *(0,1)*×*(0,1)*. Although we have not examined a dynamic adjustment process for equilibrium, we can hence suppose that the equilibrium state *(1,1)*×*(1,1)* where the community attain cooperation in both activities is more robust than the equilibrium state *(0,1)*×*(0,1)*.

In contrast to the case of G-Community, the payoff in each equilibrium in the I-Community does not differ so much from that in other equilibrium. The cooperative equilibriums *(1,1)*×*(1,1)*,*(1,0)*×*(1,0)*,*(0,1)*×*(0,1)* cannot outweigh the non-cooperative one (0,0)×(0,0) in terms of payoff in the repeated and liked game. We can hence suppose that although the equilibriums in which cooperation in activities can be attained, such equilibriums are fragile.

Concluding Remarks

This paper has proposed a model for describing a spontaneous institution as a cooperative equilibrium in infinitely repeated and linked game. We have installed the linkage of some types of activities in a community into the model.

Results in the numerical analysis have illustrated how the factors characterizing a community affect on the vitality and robustness of cooperative equilibriums.

The model analysis has been still under theoretical development. We have to extend the model so that it can be consistent with a representation of social network. We will then be able to analyze the case of spontaneous leaders in community activities, the case of asymmetric information flow among individuals, and other interesting cases.

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